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Common Fixed Point Theorems in Anti Fuzzy Metric Spaces

M. Jeyaraman[a,*], R. Pandiselvi[b], D. Poovaragavan[c]

[a] PG and Research Department of Mathematics, Raja Doraisingam Govt. Arts College,
Sivagangai, Affiliated to Alagappa University,
Karaikudi, Tamil Nadu, India, e-mail: jeya.math@gmail.com
[b] PG and Research Department of Mathematics, The Madura College,
Madurai, Tamilnadu, India. e-mail: rpselvi@gmail.com
[c] Department of Mathematics, Govt. Arts College for Women, Sivagangai,
Affiliated to Alagappa University, Karaikudi,
Tamil Nadu, India. e-mail: poovaragavan87@gmail.com.

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Abstract

This article introduces the innovative concept of anti-fuzzy metric spaces and utilizes the property (E.A.) and Common limit range property of $\mathfrak{Q}$, we demonstrate the existence and uniqueness of a common fixed point in symmetric anti fuzzy metric spaces in this study. We discuss some novel ideas for a few mappings named R-weakly commuting of type $(\mathfrak{I}_{\mathfrak{P}})$ and weakly commuting of type $(\mathfrak{I}_{\mathfrak{P}})$ on an anti fuzzy metric space. Mathematics Subject Classification 2010 (MSC) 54H25 47H10.

1. Introduction

In 2006, Mustafa and Sims [1] presented the significance of G-metric space. A few fixed point results were shown in G - metric spaces from here on out. Then again, In 2010, Sun and Yang [2] presented the idea of generalized fuzzy metric space utilizing the idea of continuous t-norm. a few interesting references on G-metric spaces [3, 4, 5, 6, 7]. In 1965, Zadeh[8] established the concept of fuzzy sets, and in the years that followed, several research articles based on the idea of fuzzy sets. Biswas [9] presented the thought of Anti Fuzzy set which was based on T_1^*. He demonstrated that an Anti Fuzzy set is only the complement of an Fuzzy Set for T_1. In 2018, Thangaraj Beaula and Beulah Mariya [10, 11] present the 2-fuzzy 2-anti equicontinuity, $\alpha - 2$ - anti standard metric, $\alpha - 2$ - anti standard bounded metric, $\alpha - 2$ - anti uniform metric are defined. We show the presence and uniqueness of a common fixed point in symmetric anti fuzzy metric spaces using property (E.A.) and $CLR_{\mathfrak{Q}}$ property.

In this article, we introduce the novel idea of anti fuzzy metric spaces and provide the new ideas for a few mappings $(\mathfrak{P}, \mathfrak{Q})$ on an anti fuzzy metric space known as weakly commuting of type $(\mathfrak{I}_{\mathfrak{P}})$ and R-weakly commuting of type $(\mathfrak{I}_{\mathfrak{P}})$.

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*Corresponding author: jeya.math@gmail.com

2. Preliminaries

Definition 2.1. A 3-tuple $(\mathfrak{U}, G_A, *)$ is called a Anti Fuzzy Metric Space(shortly AFMS) if $\mathfrak{U}$ is an arbitrary non-empty set, $*$ is a continuous t-norm, G_A is fuzzy sets on $\mathfrak{U}^3 \times (0, \infty)$ fulfilling the accompanying conditions: For every $\rho, \sigma, \sigma, a \in \mathfrak{U}$ and $\lambda, \mu > 0$.

(i) $G_A(\rho, \rho, \sigma, \lambda) < 1$ for $\rho \neq \sigma$,

(ii) $G_A(\rho, \rho, \sigma, \lambda) \leqslant G_A(\rho, \sigma, \sigma, \lambda)$ for $\sigma \neq \sigma$,

(iii) $G_A(\rho, \sigma, \sigma, \lambda) = 0$ if and only if $\rho = \sigma = \sigma$,

(iv) $G_A(\rho, \sigma, \sigma, \lambda) = G_A(p(\rho, \sigma, \sigma), \lambda)$, where p is a permutation function,

(v) $G_A(\rho, a, a, \lambda) * G_A(a, \sigma, \sigma, \mu) \geqslant G_A(\rho, \sigma, \sigma, \lambda + \mu)$,

(vi) $G_A(\rho, \sigma, \sigma, .) : (0, \infty) \to [0, 1]$ is continuous,

(vii) G_A is a non - increasing function on R^+, $\lim\limits_{\lambda \to \infty} G_A(\rho, \sigma, \sigma, \lambda) = 0$ and $\lim\limits_{\lambda \to 0} G_A(\rho, \sigma, \sigma, \lambda) = 1$.

In this case, G_A is called a AFM on $\mathfrak{U}$.

Example 2.2. Let $\mathfrak{U}$ be a non-empty set and let G be metrics on $\mathfrak{U}^3$ where t-norm is defined by $p * q = \max\{p, q\}$. For all $\rho, \sigma, \sigma \in \mathfrak{U}$ and $\lambda > 0, G_A(\rho, \sigma, \sigma, \lambda) = \frac{G(\rho,\sigma,\sigma)}{\lambda + G(\rho,\sigma,\sigma)}$. Then $(\mathfrak{U}, G_A, *)$ is a AFMS.

Definition 2.3. Let $(\mathfrak{U}, G_A, *)$ be a AFMS.

(i) A sequence $\{\rho_n\}$ in $\mathfrak{U}$ is called convergent to ρ if and $\lim\limits_{n \to \infty} G_A(\rho_n, \rho_n, \rho, \lambda) = 0$

(ii) A sequence $\{\rho_n\}$ in $\mathfrak{U}$ is called a Cauchy sequence if $\lim\limits_{n,m \to \infty} G_A(\rho_n, \rho_n, \rho_m, \lambda) = 0$, that is, for any $\epsilon > 0$ and for each $\lambda > 0$, there exists $n_0 \in \mathbb{N}$ such that $G_A(\rho_n, \rho_n, \rho_m, \lambda) < \epsilon$ for every $n, m \geqslant n_0$.

(iii) A AFMS $(\mathfrak{U}, G_A, *)$ is called complete if every Cauchy sequence in $\mathfrak{U}$ is convergent.

Definition 2.4. Let $(\mathfrak{U}, G_A, *)$ be a AFMS. If the conditions $\lim\limits_{n \to \infty} G_A(\rho_n, \sigma_n, \sigma_n, \lambda_n) = G_A(\rho, \sigma, \sigma, \lambda)$ are fulfilled at whatever point $\lim\limits_{n \to \infty} \rho_n = \rho$, $\lim\limits_{n \to \infty} \sigma_n = \sigma$, $\lim\limits_{n \to \infty} \sigma_n = \sigma$ and $\lim\limits_{n \to \infty} G_A(\rho, \sigma, \sigma, \lambda_n) = G_A(\rho, \sigma, \sigma, \lambda)$, then G_A is called convergent functions on $\mathfrak{U}^3 \times (0, \infty)$.

Lemma 2.5. *Let $(\mathfrak{U}, G_A, *)$ be a AFMS. Then G_A is continuous function on $\mathfrak{U}^3 \times (0, \infty)$.*

Proof. Since $\lim\limits_{n \to \infty} \rho_n = \rho$, $\lim\limits_{n \to \infty} \sigma_n = \sigma$, $\lim\limits_{n \to \infty} \sigma_n = \sigma$ and $\lim\limits_{n \to \infty} G_A(\rho, \sigma, \varepsilon, \lambda_n) = G_A(\rho, \sigma, \sigma, \lambda)$, there is $n_0 \in \mathbb{N}$ such that $|\lambda - \lambda_n| < \epsilon$ and $|\lambda - \lambda_n| > \delta$ for every $n \geqslant n_0$ and $\epsilon < \frac{\lambda}{2}$ and $\delta > \frac{\lambda}{2}$.

Clearly $G_A(\rho,\sigma,\sigma,\lambda)$ is decreasing with respect to λ. So, we have

$$\begin{aligned}
G_A(\rho_n,\sigma_n,\sigma_n,\lambda) \leqslant G_A(\rho_n,\sigma_n,\sigma_n,\lambda-\delta) &\leqslant G_A\left(\rho_n,\rho,\rho,\frac{\delta}{3}\right) * G_A\left(\rho,\sigma_n,\sigma_n,\lambda-\frac{4\delta}{3}\right)\\
&\leqslant G_A\left(\rho_n,\rho,\rho,\frac{\delta}{3}\right) * G_A\left(\sigma_n,\sigma,\sigma,\frac{\delta}{3}\right) * G_A\left(\sigma,\rho,\sigma_n,\lambda-\frac{5\delta}{3}\right)\\
&\leqslant G_A\left(\rho_n,\rho,\rho,\frac{\delta}{3}\right) * G_A\left(\sigma_n,\sigma,\sigma,\frac{\delta}{3}\right) * G_A\left(\sigma_n,\sigma,\sigma,\frac{\delta}{3}\right) * G_A(\sigma,\sigma,\sigma,\lambda-2\delta).\\
G_A(\rho,\sigma,\sigma,\lambda+2\delta) \leqslant G_A(\rho,\sigma,\sigma,\lambda_n+\delta) &\leqslant G_A\left(\rho,\rho_n,\rho_n,\frac{\delta}{3}\right) * G_A\left(\rho_n,\sigma,\sigma,\lambda_n+\frac{2\delta}{3}\right)\\
&\leqslant G_A\left(\rho,\rho_n,\rho_n,\frac{\delta}{3}\right) * G_A\left(\sigma,\sigma_n,\sigma_n,\frac{\delta}{3}\right) * G_A\left(\sigma_n,\rho_n,\sigma,\lambda_n+\frac{\delta}{3}\right)\\
&\leqslant G_A\left(\rho,\rho_n,\rho_n,\frac{\delta}{3}\right) * G_A\left(\sigma,\sigma_n,\sigma_n,\frac{\delta}{3}\right) * G_A\left(\sigma,\sigma_n,\sigma_n,\frac{\delta}{3}\right) * G_A(\sigma,\sigma,\rho,\lambda_n)
\end{aligned}$$

Let $n \to \infty$. By continuity of the functions G_A, with respect to λ, we get

$$G_A(\rho,\sigma,\sigma,\lambda+2\delta) \leqslant G_A(\sigma,\sigma,\rho,\lambda) \leqslant G_A(\sigma,\sigma,\rho,\lambda-2\delta).$$

Therefore G_A is continuous functions on $\mathfrak{U}^3 \times (0,\infty)$. □

3. Weakly Commuting of Type $(\mathfrak{J}_{\mathfrak{F}})$

Definition 3.1. A couple of self mapping $(\mathfrak{P},\mathfrak{Q})$ of AFMS $(\mathfrak{U},G_A,*)$ is called Weakly Commuting of type $(\mathfrak{J}_{\mathfrak{P}})$ (shortly $WC(\mathfrak{J}_{\mathfrak{P}})$) if $G_A(\mathfrak{P}\mathfrak{Q}\rho,\mathfrak{Q}\mathfrak{P}\rho,\mathfrak{P}\mathfrak{P}\rho,\lambda) \leqslant G_A(\mathfrak{P}\rho,\mathfrak{Q}\rho,\mathfrak{P}\rho,\lambda)$, for every $\rho \in \mathfrak{U}$ with $\lambda > 0$.

Definition 3.2. A couple of self mapping $(\mathfrak{P},\mathfrak{Q})$ of AFMS $(\mathfrak{U},G_A,*)$ is called R - WC $(\mathfrak{J}_{\mathfrak{P}})$ on the off chance that there exists $R > 0$ such that
$G_A(\mathfrak{P}\mathfrak{Q}\rho,\mathfrak{Q}\mathfrak{P}\rho,\mathfrak{P}\mathfrak{P}\rho,\lambda) \leqslant G_A(\mathfrak{P}\rho,\mathfrak{Q}\rho,\mathfrak{P}\rho,\lambda/R)$, for every $\rho \in \mathfrak{U}$ with $\lambda > 0$.

Remark 3.3. If exchange $\mathfrak{P}$ and $\mathfrak{Q}$ in Definitions (3.1) and Definitions (3.2), the couple of self mapping $(\mathfrak{Q},\mathfrak{P})$, of AFMS $(\mathfrak{U},G_A,*)$ is called WC $(\mathfrak{J}_{\mathfrak{Q}})$ or R - WC $(\mathfrak{J}_{\mathfrak{Q}})$, respectively.

Example 3.4. Let $\mathfrak{U} = [0,1]$ be AFMS defined by $G_A(\rho,\sigma,\sigma,\lambda) = \frac{|\rho-\sigma|+|\sigma-\sigma|+|\sigma-\rho|}{\lambda+|\rho-\sigma|+|\sigma-\sigma|+|\sigma-\rho|}$ for every $\rho,\sigma,\sigma \in \mathfrak{U}$ with $\lambda > 0$. Define $\mathfrak{P},\mathfrak{Q} : \mathfrak{U} \to \mathfrak{U}$ by $\mathfrak{P}\rho = \frac{\rho^2}{4}, \mathfrak{Q}\rho = \rho^2$, for all $\rho \in \mathfrak{U}$. Clearly, $\rho = 0$ is the one and only coincidence point of $\mathfrak{P}$ and $\mathfrak{Q}$.
Also, $\mathfrak{P}\left(\mathfrak{Q}\left(\frac{1}{2}\right)\right) = \mathfrak{P}\left(\frac{1}{2}\right) = \frac{1}{2}$ and $\mathfrak{Q}\left(\mathfrak{P}\left(\frac{1}{2}\right)\right) = \mathfrak{Q}\left(\frac{1}{2}\right) = \frac{1}{2}$. So, $\mathfrak{P}$ and $\mathfrak{Q}$ are WC. It should be noted that $G_A(\mathfrak{P}\mathfrak{Q}\rho,\mathfrak{Q}\mathfrak{P}\rho,\mathfrak{P}\mathfrak{P}\rho,\lambda) \leqslant G_A(\mathfrak{P}\rho,\mathfrak{Q}\rho,\mathfrak{P}\rho,\lambda)$,for all $\rho \in \mathfrak{U}$ and $\lambda > 0$. Then the couple $(\mathfrak{P},\mathfrak{Q})$ is WC $(\mathfrak{J}_{\mathfrak{P}})$ but not WC $(\mathfrak{J}_{\mathfrak{Q}})$.

Example 3.5. Let $\mathfrak{U} = [0,2]$ be AFMS. Define $\mathfrak{P}\rho = 2-\rho, \mathfrak{Q}\rho = \rho$, then the pair $(\mathfrak{P},\mathfrak{Q})$ is WC $(\mathfrak{J}_{\mathfrak{P}})$ and R - WC $(\mathfrak{J}_{\mathfrak{P}})$.

Lemma 3.6. *If $\mathfrak{P}$ and $\mathfrak{Q}$ are WC $(\mathfrak{J}_{\mathfrak{P}})$ or R - WC $(\mathfrak{J}_{\mathfrak{P}})$, then $\mathfrak{P}$ and $\mathfrak{Q}$ are WC.*

Proof. Consider the coincidence point ρ of $\mathfrak{P}$ and $\mathfrak{Q}$, i.e., $\mathfrak{P}\rho = \mathfrak{Q}\rho$. If the couple $(\mathfrak{P},\mathfrak{Q})$ of AFMS $(\mathfrak{U},G_A,*)$ is WC $(\mathfrak{J}_{\mathfrak{P}})$, we have

$$G_A(\mathfrak{P}\mathfrak{Q}\rho,\mathfrak{Q}\mathfrak{P}\rho,\mathfrak{P}\mathfrak{Q}\rho,\lambda) = G_A(\mathfrak{P}\mathfrak{Q}\rho,\mathfrak{Q}\mathfrak{P}\rho,\mathfrak{P}\mathfrak{P}\rho,\lambda) \leqslant G_A(\mathfrak{P}\rho,\mathfrak{Q}\rho,\mathfrak{Q}\rho,\lambda) \leqslant 0.$$

It follows that $\mathfrak{P}\mathfrak{Q}\rho = \mathfrak{Q}\mathfrak{P}\rho$, and then get their coincidence point. Correspondingly, if the couple $(\mathfrak{P}, \mathfrak{Q})$ of AFMS $(\mathfrak{U}, G_A, *)$ is R- WC $(\mathfrak{J}_{\mathfrak{P}})$, we have, for every $\rho \in \mathfrak{U}$,

$$G_A(\mathfrak{P}\mathfrak{Q}\rho, \mathfrak{Q}\mathfrak{P}\rho, \mathfrak{P}\mathfrak{Q}\rho, \lambda) \leqslant G_A(\mathfrak{P}\mathfrak{Q}\rho, \mathfrak{Q}\mathfrak{P}\rho, \mathfrak{P}\mathfrak{P}\rho, \lambda) \leqslant G_A(\mathfrak{P}\rho, \mathfrak{Q}\rho, \mathfrak{P}\rho, \lambda/R) = 0.$$

Thus $\mathfrak{P}\mathfrak{Q}\rho = \mathfrak{Q}\mathfrak{P}\rho$, which implies that $\mathfrak{P}$ and $\mathfrak{Q}$ are WC. □

Definition 3.7. A couple of self mapping $(\mathfrak{P}, \mathfrak{Q})$ on $\mathfrak{U}$ is called to fulfill the property (E.A.) if there exists a sequence $\{\rho_n\}$ such that $\lim_{n\to\infty} \mathfrak{P}\rho_n = \lim_{n\to\infty} \mathfrak{Q}\rho_n = \sigma$ for all $\sigma \in \mathfrak{U}$.

Definition 3.8. A couple of self mapping $(\mathfrak{P}, \mathfrak{Q})$ on $\mathfrak{U}$ is called to fulfill the $CLR_{\mathfrak{Q}}$ property if there exists a sequence $\{\rho_n\}$ such that $\lim_{n\to\infty} \mathfrak{P}\rho_n = \lim_{n\to\infty} \mathfrak{Q}\rho_n = \mathfrak{Q}\sigma$ for all $\sigma \in \mathfrak{U}$. For demonstrating our fundamental outcomes, we utilize following connection: Define $\Phi = \{\phi : [0, \infty) \to [0, \infty)\}$ and each $\phi \in \Phi$ fulfills the accompanying conditions:
$(\phi - 1)$: ϕ is strictly non-decreasing.
$(\phi - 2)$: ϕ is upper semi-continuous from the right.
$(\phi - 3)$:$\sum_{n=0}^{\infty} \phi^n(t) < \infty$, for every $t > 0$.

Lemma 3.9. *Let* $(\mathfrak{U}, G_A, *)$ *be a AFMS. Not likely to be* $\phi \in \Phi$ *such that* $G_A(\rho, \sigma, \sigma, \phi(\lambda)) \leqslant G_A(\rho, \sigma, \sigma, \lambda)$, *for every* $\lambda > 0$, *then* $\rho = \sigma = \sigma$.

Lemma 3.10. *Let* $(\mathfrak{U}, G_A, *)$ *be a AFMS. If we define* $E_\mu : \mathfrak{U} \times \mathfrak{U} \times \mathfrak{U} \to [0, \infty)$ *by*

$$E_\mu(\rho, \sigma, \sigma) = \sup\{\lambda > 0, G_A(\rho, \sigma, \sigma, \lambda) < \mu\} \tag{3.10.1}$$

for all $\mu \in (0, 1]$ *and* $\rho, \sigma, \sigma \in \mathfrak{U}$, *then we have for each* $\mu \in (0, 1]$, *there exists* $\upsilon \in (0, 1]$ *such that* $E_\mu(\rho_1, \rho_1, \rho_n) \geqslant \sum_{i=1}^{n-1} E_\upsilon(\rho_i, \rho_i, \rho_{i+1})$, *for all* $\rho_1, \ldots, \rho_n \in \mathfrak{U}$.

Proof. For any $\mu \in (0, 1]$, let $\upsilon \in (0, 1]$ and $\upsilon < \lambda$, and for any $\delta > 0$, we have

$$\begin{aligned} G_A\left(\rho_1, \rho_1, \rho_n, \sum_{i=1}^{n-1} E_\upsilon(\rho_i, \rho_i, \rho_{i+1}) + (n-1)\delta\right) \\ &\leqslant G_A\Big(\rho_1, \rho_1, \rho_2, E_\upsilon(\rho_1, \rho_1, \rho_2 + \delta\Big) * G_A\Big(\rho_2, \rho_2, \rho_3, E_\upsilon(\rho_2, \rho_2, \rho_3 + \delta\Big) * \cdots \\ &\qquad * G_A\Big(\rho_{n-1}, \rho_{n-1}, \rho_n, E_\upsilon(\rho_{n-1}, \rho_{n-1}, \rho_n + \delta\Big) \\ &\leqslant \max\{\upsilon, \upsilon, \ldots, \upsilon\} \leqslant \upsilon, \end{aligned}$$

which implies, by (3.10.1)

$$E_\mu(\rho_1, \rho_1, \rho_n) \geqslant E_\mu(\rho_1, \rho_1, \rho_2) + E_\mu(\rho_2, \rho_2, \rho_3) + \cdots + E_\mu(\rho_{n-1}, \rho_{n-1}, \rho_n) + (n-1)\delta.$$

Since $\delta > 0$ is arbitrary, we have

$$E_\mu(\rho_1, \rho_1, \rho_n) \geqslant E_\mu(\rho_1, \rho_1, \rho_2) + E_\mu(\rho_2, \rho_2, \rho_3) + \cdots + E_\mu(\rho_{n-1}, \rho_{n-1}, \rho_n).$$

□

Lemma 3.11. *Let* $(\mathfrak{U}, G_A, *)$ *be a AFMS and* $\{\rho_n\}$ *be any sequence in* $\mathfrak{U}$. *If so* $\phi \in \Phi$ *such that* $G_A(\rho_n, \rho_n, \rho_{n+1}, \phi(\lambda)) \leqslant G_A(\rho_{n-1}, \rho_{n-1}, \rho_n, \lambda) * G_A(\rho_n, \rho_n, \rho_{n+1}, \lambda)$, *for every* $\lambda > 0$ *with* $n = 1, 2, \ldots$. *Then* $\{\rho_n\}$ *is a Cauchy sequence in* $\mathfrak{U}$.

Proof. Consider the sequence $\{E_\mu(\rho,\sigma,\sigma)\}_{\mu\in(0,1]}$. For each $\mu \in (0,1]$ and $n \in \mathbb{N}$, putting $a_n = E_\mu(\rho_{n-1},\rho_{n-1},\rho_n)$, we will prove that

$$a_{n+1} \geqslant \phi(a_n), \quad n \in \mathbb{N}. \tag{3.11.1}$$

Since ϕ is upper semi-continuous from right, for given $\varepsilon > 0$ and each a_n, there exists $\mathfrak{p}_n < a_n$ such that $\phi(\mathfrak{p}_n) > \phi(a_n) + \varepsilon$. From the definition of E_μ, it follows from $\mathfrak{p}_n < a_n = E_\mu(\rho_{n-1},\rho_{n-1},\rho_n)$ that $G_A(\rho_{n-1},\rho_{n-1},\rho_n,\mathfrak{p}_n) < \mu$ for all $n \in \mathbb{N}$.
Thus,

$$\begin{aligned}
&G_A\Big(\rho_n,\rho_n,\rho_{n+1},\phi\Big(\max(\mathfrak{p}_n,\mathfrak{p}_{n+1})\Big)\Big)\\
&\qquad \leqslant G_A\Big(\rho_{n-1},\rho_{n-1},\rho_n,\max(\mathfrak{p}_n,\mathfrak{p}_{n+1})\Big) * G_A\Big(\rho_n,\rho_n,\rho_{n+1},\max(\mathfrak{p}_n,\mathfrak{p}_{n+1})\Big)\\
&\qquad \leqslant G_A\Big(\rho_{n-1},\rho_{n-1},\rho_n,\mathfrak{p}_n\Big) * G_A\Big(\rho_n,\rho_n,\rho_{n+1},\mathfrak{p}_{n+1}\Big)\\
&\qquad < \mu.
\end{aligned}$$

Again by (3.10.1), we get

$$\begin{aligned}
E_\mu(\rho_n,\rho_n,\rho_{n+1}) &\geqslant \phi\Big(\max(\mathfrak{p}_n,\mathfrak{p}_{n+1})\Big) = \max\Big\{\phi(\mathfrak{p}_n),\phi(\mathfrak{p}_{n+1})\Big\}\\
&\geqslant \max\Big\{\phi(a_n),\phi(a_{n+1})\Big\} + \varepsilon.
\end{aligned}$$

By the arbitrariness of ε, we have

$$a_{n+1} = E_\mu(\rho_n,\rho_n,\rho_{n+1}) \geqslant \max\Big\{\phi(a_n),\phi(a_{n+1})\Big\}. \tag{3.11.2}$$

So,we can infer that $a_{n+1} \geqslant \phi(a_n)$. If not, then by (3.11.2), we have $a_{n+1} \geqslant \phi(a_{n+1}) \geqslant a_{n+1}$, which is a contradiction. Hence, (3.11.2) implies that $a_{n+1} \geqslant \phi(a_n)$, and (3.11.1) is proved. Again and again using (3.11.1), we get

$$E_\mu(\rho_n,\rho_n,\rho_{n+1}) \geqslant \phi\Big(E_\mu(\rho_{n-1},\rho_{n-1},\rho_n)\Big) \geqslant \cdots \geqslant \phi^n\Big(E_\mu(\rho_0,\rho_0,\rho_1)\Big) \quad \text{for all } n \in \mathbb{N}.$$

By lemma(3.10) for each $\mu \in (0,1]$, there exists $\upsilon \in (0,\mu]$ such that

$$E_\mu(\rho_n,\rho_n,\rho_m) \geqslant \sum_{i=n}^{m-1} E_\upsilon(\rho_i,\rho_i,\rho_{i+1}), \quad m,n \in \mathbb{N} \text{ with } m > n. \tag{3.11.3}$$

Since $\phi \in \Phi$, by condition $(\phi - 3)$ we have $\sum_{n=0}^{\infty} \phi^n E_\mu(\rho_0,\rho_0,\rho_1) < \infty$. So, for given $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $\sum_{i=n_0}^{\infty} \phi^i E_\mu(\rho_0,\rho_0,\rho_1) > \varepsilon$. Thus, it follows from (3.11.3) that

$$E_\mu(\rho_n,\rho_n,\rho_m) \geqslant \sum_{i=n}^{\infty} \phi^i E_\mu(\rho_0,\rho_0,\rho_1) > \varepsilon, \quad \text{for all } \ n \geqslant n_0,$$

which implies that $G_A(\rho_n,\rho_n,\rho_m,\varepsilon) < \mu$ for all $m,n \in \mathbb{N}$ with $m > n \geqslant n_0$. Therefore, ρ_n is a Cauchy sequence in $\mathfrak{U}$. □

4. Main Results

Theorem 4.1. *Let* $(\mathfrak{U}, G_A, *)$ *be a symmetric AFMS and the mappings* $\mathfrak{P}, \mathfrak{Q} : \mathfrak{U} \to \mathfrak{U}$ *fulfills the accompanying conditions:*
(4.1.1) $\mathfrak{P}$ *and* $\mathfrak{Q}$ *are WC* $(\mathfrak{J}_{\mathfrak{P}})$*,*
(4.1.2) $\mathfrak{P}(\mathfrak{U}) \subseteq \mathfrak{Q}(\mathfrak{U})$*,*
(4.1.3) $\mathfrak{Q}(\mathfrak{U})$ *is a complete subspace of* $\mathfrak{U}$*,*
(4.1.4) there exists a $\phi \in \Phi$ *such that, for every* $\rho, \sigma, \sigma \in \mathfrak{U}$ *with* $\lambda > 0$*,*

$$G_A(\mathfrak{P}\rho, \mathfrak{P}\sigma, \mathfrak{P}\sigma, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}\rho, \mathfrak{Q}\rho, \mathfrak{P}\rho, \lambda) * G_A(G\sigma, \mathfrak{Q}\sigma, \mathfrak{P}\sigma, \lambda) * G_A(\mathfrak{Q}\sigma, \mathfrak{Q}\sigma, \mathfrak{P}\sigma, \lambda).$$

Then $\mathfrak{P}$ *and* $\mathfrak{Q}$ *to the common fixed point.*

Proof. Choose $\rho_0, \rho_1, \rho_2 \in \mathfrak{U}$ be such that $\mathfrak{P}\rho_0 = \mathfrak{Q}\rho_1$ and $\mathfrak{P}\rho_1 = \mathfrak{Q}\rho_2$.
Then by mathematical induction, establish a sequence $\{\rho_n\} \in \mathfrak{U}$ as follows:
$\sigma_n = \mathfrak{P}\rho_n = \mathfrak{Q}\rho_{n+1}, n \in \mathbb{N}$. To prove $\{\sigma_n\}$ is a Cauchy sequence in $\mathfrak{U}$.

$$\begin{aligned}
&G_A(\sigma_n, \sigma_n, \sigma_{n+1}, \phi(\lambda)) = G_A(\mathfrak{P}\rho_n, \mathfrak{P}\rho_n, \mathfrak{P}\rho_{n+1}, \phi(\lambda)) \\
&\leqslant G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{P}\rho_n, \lambda) * G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{P}\rho_n, \lambda) * G_A(\mathfrak{Q}\rho_{n+1}, \mathfrak{Q}\rho_{n+1}, \mathfrak{P}\rho_{n+1}, \lambda) \\
&\leqslant G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{Q}\rho_{n+1}, \lambda) * G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{Q}\rho_{n+1}, \lambda) * G_A(\mathfrak{Q}\rho_{n+1}, \mathfrak{Q}\rho_{n+1}, \mathfrak{Q}\rho_{n+2}, \lambda) \\
&= G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{Q}\rho_{n+1}, \lambda) * G_A(\mathfrak{Q}\rho_{n+1}, \mathfrak{Q}\rho_{n+1}, \mathfrak{Q}\rho_{n+2}, \lambda).
\end{aligned}$$

This gives $G_A(\sigma_n, \sigma_n, \sigma_{n+1}, \phi(\lambda)) \leqslant G_A(\sigma_{n-1}, \sigma_{n-1}, \sigma_n, \lambda) * G_A(\sigma_n, \sigma_n, \sigma_{n+1}, \lambda)$.
By Lemma (3.11), the sequence $\{\sigma_n\}$ is a Cauchy sequence. Since $\sigma_n = \mathfrak{Q}\rho_{n+1}, \{\mathfrak{Q}\rho_{n+1}\}$ is also a Cauchy sequence in $G(\mathfrak{U})$. By (4.1.3) hypotheses, $\mathfrak{Q}(\mathfrak{U})$ is complete, and then, at that point, there exists $u \in \mathfrak{Q}(\mathfrak{U})$ to such an extent that $\lim\limits_{n\to\infty} \mathfrak{Q}\rho_n = \lim\limits_{n\to\infty} \mathfrak{P}\rho_n = u$.
Presently $u \in \mathfrak{Q}(\mathfrak{U})$, so there exists $p \in \mathfrak{U}$ to such an extent that $u = p$.
Hence $\lim\limits_{n\to\infty} \mathfrak{P}\rho_n = \lim\limits_{n\to\infty} \mathfrak{Q}\rho_n = \mathfrak{Q}p$. We will exhibit that $\mathfrak{Q}p = \mathfrak{P}p$.

$$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{P}\rho_n, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{P}\rho_n, \lambda).$$

Taking limit as $n \to \infty$, we have
$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{Q}p, \lambda)$ or
$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda)$.
Since AFMS is symmetric, we have

$$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) = G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \lambda)$$

which suggest $\mathfrak{P}p = \mathfrak{Q}p$ (by Lemma 3.9).
Since the couple $(\mathfrak{F}, \mathfrak{G})$ is WC $(\mathfrak{J}_{\mathfrak{F}})$, then

$$G_A(\mathfrak{P}\mathfrak{Q}p, \mathfrak{Q}\mathfrak{P}p, \mathfrak{P}\mathfrak{P}p, \phi(\lambda)) \leqslant G_A(\mathfrak{P}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) = 0,$$

imply $\mathfrak{P}\mathfrak{P}p = \mathfrak{P}\mathfrak{Q}p = \mathfrak{Q}\mathfrak{P}p = \mathfrak{Q}\mathfrak{Q}p$. Hence $\mathfrak{P}u = \mathfrak{P}\mathfrak{Q}p = \mathfrak{Q}\mathfrak{P}p = \mathfrak{Q}u$.
Finally, that's what we show $\mathfrak{Q}p = u$ is a common fixed point of $\mathfrak{P}$ and $\mathfrak{Q}$. Assume $u \neq \mathfrak{P}u$, then
$G_A(\mathfrak{P}u, \mathfrak{P}p, \mathfrak{P}p, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}u, \mathfrak{Q}u, \mathfrak{P}u, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda)$,
$G_A(\mathfrak{P}u, \mathfrak{P}p, \mathfrak{P}p, \phi(\lambda)) \leqslant G_A(\mathfrak{P}u, \mathfrak{P}u, \mathfrak{P}u, \lambda) * G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{P}p, \lambda)$,
$G_A(\mathfrak{P}u, u, u, \phi(\lambda)) \leqslant 0 * 0 * 0 = 0$, an inconsistency. Hence, $\mathfrak{Q}u = \mathfrak{P}u = u$.

To exhibit the uniqueness, assume we have u and v with the end goal that $u \neq v, \mathfrak{P}u = \mathfrak{Q}u = u$ and $\mathfrak{P}v = \mathfrak{Q}v = v$, of course utilizing condition (4.1.4), we have,

$$\begin{aligned}G_A(u, v, v, \phi(\lambda)) &= G_A(\mathfrak{P}u, \mathfrak{P}v, \mathfrak{P}v, \phi(\lambda))\\ &\leqslant G_A(\mathfrak{Q}u, \mathfrak{Q}u, \mathfrak{P}u, \lambda) * G_A(\mathfrak{Q}v, \mathfrak{Q}v, \mathfrak{P}v, \lambda) * G_A(\mathfrak{Q}v, \mathfrak{Q}v, \mathfrak{P}v, \lambda)\\ &= 0 * 0 * 0 = 0.\end{aligned}$$

Subsequently $G_A(u, v, v, \phi(\lambda)) \leqslant 0$, which gives inconsistency. Henceforth $u = v$. In this way u is a unique common fixed point. ☐

Example 4.2. Let $\mathfrak{U} = [0, 1]$ be AFMS. Define $\mathfrak{P}, \mathfrak{Q} : \mathfrak{U} \to \mathfrak{U}$ by $\mathfrak{P}(\rho) = \frac{\rho^2}{4}, \mathfrak{Q}(\rho) = \rho^2, \rho \in \mathfrak{U}$. That's what we see $\rho = 0$ is one and only coincidence point. $\mathfrak{P}\left(\mathfrak{Q}\left(\frac{1}{2}\right)\right) = \mathfrak{P}\left(\frac{1}{2}\right) = \frac{1}{2}$ and $\mathfrak{Q}\left(\mathfrak{P}\left(\frac{1}{2}\right)\right) = \mathfrak{Q}\left(\frac{1}{2}\right) = \frac{1}{2}$.
So, $\mathfrak{P}$ and $\mathfrak{Q}$ are weakly commuting. Also $G_A(\mathfrak{P}\mathfrak{Q}\rho, \mathfrak{Q}\mathfrak{P}\rho, \mathfrak{P}\mathfrak{P}\rho, \lambda) \leqslant G_A(\mathfrak{P}\rho, \mathfrak{Q}\rho, \mathfrak{P}\rho, \lambda)$. Then the couple $(\mathfrak{P}, \mathfrak{Q})$ WC $(\mathfrak{J}_{\mathfrak{P}})$ but not $WC(\mathfrak{J}_{\mathfrak{Q}})$.

Example 4.3. Let $\mathfrak{U} = [-1, 1]$ be AFMS. Let $\phi(\lambda) = \frac{\lambda}{2}$ and the self mapping is define $\mathfrak{P}, \mathfrak{Q} : \mathfrak{U} \to \mathfrak{U}$ by $\mathfrak{P}(\rho) = \frac{\rho}{6}, \mathfrak{Q}(\rho) = \frac{\rho}{2}(\rho + 1), \rho \in \mathfrak{U}$. Clearly $\rho = 0$ is the one and only coincidence point, and, $\mathfrak{P}$ and $\mathfrak{Q}$ are weakly compatible. Let $\{\rho_n = \frac{1}{n}\}$ be a sequence with the end goal that $G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{P}\rho_n, \phi(\lambda)) \leqslant G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \lambda)$, where p is a coincidence point.
Then the couple $(\mathfrak{P}, \mathfrak{Q})$ is WC $(\mathfrak{J}_{\mathfrak{P}})$, and, $\mathfrak{P}$ and $\mathfrak{Q}$ have a one and only common fixed point.

Corollary 4.4. *Theorem (4.1) remains true if we replace WC $(\mathfrak{J}_{\mathfrak{P}})$ by WC $(\mathfrak{J}_{\mathfrak{Q}})$ or by R - WC $(\mathfrak{J}_{\mathfrak{Q}})$.*

Theorem 4.5. *Let $(\mathfrak{U}, G_A, *)$ be a symmetric AFMS and suppose mappings $\mathfrak{P}, \mathfrak{Q} : \mathfrak{U} \to \mathfrak{U}$ are WC $(\mathfrak{J}_{\mathfrak{F}})$ fulfilling the accompanying conditions:*
(4.5.1) $\mathfrak{P}$ and $\mathfrak{Q}$ satisfy the property (E.A.),
(4.5.2) $\mathfrak{Q}(\mathfrak{U})$ is a closed subspace of $\mathfrak{U}$,
(4.5.3) there exists $\phi \in \Phi$ such that for every $\rho, \sigma, \sigma \in \mathfrak{U}$ with $\lambda > 0$,

$$G_A(\mathfrak{P}\rho, \mathfrak{P}\sigma, \mathfrak{P}\sigma, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}\rho, \mathfrak{Q}\rho, \mathfrak{P}\rho, \lambda) * G_A(\mathfrak{Q}\sigma, \mathfrak{Q}\sigma, \mathfrak{P}\sigma, \lambda) * G_A(\mathfrak{Q}\sigma, \mathfrak{Q}\sigma, \mathfrak{P}\sigma, \lambda).$$

Then $\mathfrak{P}$ and $\mathfrak{Q}$ have a unique common fixed point.

Proof. The mappings $\mathfrak{P}$ and $\mathfrak{Q}$ fulfill the property (E.A.), then there exists a sequence $\{\rho_n\} \in \mathfrak{U}$ fulfilling $\lim_{n\to\infty} \mathfrak{Q}\rho_n = u = \lim_{n\to\infty} \mathfrak{P}\rho_n$, for any $u \in \mathfrak{U}$. Since $\mathfrak{Q}(\mathfrak{U})$ is a closed subspace of $\mathfrak{U}$ and $\lim_{n\to\infty} \mathfrak{Q}\rho_n = u$, then there exists $p \in \mathfrak{U}$ such that $u = \mathfrak{Q}p$.
Additionally $\lim_{n\to\infty} \mathfrak{Q}\rho_n = \mathfrak{Q}p = \lim_{n\to\infty} \mathfrak{P}\rho_n$. We will demonstrate that, $\mathfrak{P}p = \mathfrak{Q}p$.

$$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{P}\rho_n, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}\rho_n, \mathfrak{Q}\rho_n, \mathfrak{P}\rho_n, \lambda),$$

taking limit as $n \to \infty$, we have
$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda) * G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{Q}p, \lambda)$
$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \phi(\lambda)) \leqslant G_A(\mathfrak{Q}p, \mathfrak{Q}p, \mathfrak{P}p, \lambda)$. Since AFMS is symmetric,
$G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \phi(\lambda)) \leqslant G_A(\mathfrak{P}p, \mathfrak{P}p, \mathfrak{Q}p, \lambda)$,
which suggests $\mathfrak{P}p = \mathfrak{Q}p = u$.

Since the pair $(\mathfrak{P},\mathfrak{Q})$ is WC $(\mathfrak{J}_{\mathfrak{F}})$, then
$G_A(\mathfrak{P}\mathfrak{Q}p,\mathfrak{Q}\mathfrak{P}p,\mathfrak{P}\mathfrak{P}p,\phi(\lambda)) \leqslant G_A(\mathfrak{P}p,p,\mathfrak{P}p,\lambda) = 0$,
which imply that $\mathfrak{P}\mathfrak{P}p = \mathfrak{P}\mathfrak{Q}p = \mathfrak{Q}\mathfrak{P}p = \mathfrak{Q}\mathfrak{Q}p$. Hence $\mathfrak{P}u = \mathfrak{P}\mathfrak{Q}p = \mathfrak{Q}\mathfrak{P}p = \mathfrak{Q}u$.
Presently, we will show that $\mathfrak{P}p = u$ is a common fixed point of $\mathfrak{P}$ and $\mathfrak{g}$.
Assume $\mathfrak{P}u \neq u$, then

$$\begin{aligned} G_A(\mathfrak{P}u,u,u,\phi(\lambda)) &= G_A(\mathfrak{P}u,\mathfrak{P}p,\mathfrak{P}p,\phi(\lambda)) \\ &\leqslant G_A(\mathfrak{Q}u,\mathfrak{Q}u,\mathfrak{P}u,\lambda) * G_A(\mathfrak{Q}p,\mathfrak{Q}p,\mathfrak{P}p,\lambda) * G_A(\mathfrak{Q}p,\mathfrak{Q}p,\mathfrak{P}p,\lambda) \\ &= G_A(\mathfrak{P}u,\mathfrak{P}u,\mathfrak{P}u,\lambda) * 0 * 0 = 0 * 0 * 0 = 0, \text{a contradiction.} \end{aligned}$$

Hence $\mathfrak{P}u = u = \mathfrak{Q}u$.
To demonstrate the uniqueness, assume u and v to such an extent that $u \neq v, \mathfrak{P}u = \mathfrak{Q}u = u$ and $\mathfrak{P}v = \mathfrak{Q}v = v$, on the other hand utilizing condition (4.5.3), we have,

$$\begin{aligned} G_A(u,v,v,\phi(\lambda)) &= G_A(\mathfrak{P}u,\mathfrak{P}v,\mathfrak{P}v,\phi(\lambda)) \\ &\leqslant G_A(\mathfrak{Q}u,\mathfrak{Q}u,\mathfrak{P}u,\lambda) * G_A(\mathfrak{Q}v,\mathfrak{Q}v,\mathfrak{P}v,\lambda) * G_A(\mathfrak{Q}v,\mathfrak{Q}v,\mathfrak{P}v,\lambda) \\ &= 0 * 0 * 0 = 0, \text{which is a contradiction.} \end{aligned}$$

Thus $u = v$. Therefore u is a one and only common fixed point of $\mathfrak{P}$ and $\mathfrak{Q}$. □

Theorem 4.6. *Let* $(\mathfrak{U}, G_A, *)$ *be a symmetric AFMS and suppose the self mappings* $\mathfrak{P},\mathfrak{Q} : \mathfrak{U} \to \mathfrak{U}$ *are WC* $(\mathfrak{J}_{\mathfrak{F}})$ *fulfilling the accompanying conditions:*
(4.6.1) $\mathfrak{P}$ *and* $\mathfrak{Q}$ *satisfy* $CLR_{\mathfrak{Q}}$ *property,*
(4.6.2) there exists $\phi \in \Phi$ *to such an extent that for each* $\rho,\sigma,\sigma \in \mathfrak{U}$ *with* $\lambda > 0$,
$G_A(\mathfrak{P}\rho,\mathfrak{P}\sigma,\mathfrak{P}\sigma,\phi(\lambda)) \leqslant G_A(\mathfrak{Q}\rho,\mathfrak{Q}\rho,\mathfrak{P}\rho,\lambda) * G_A(\mathfrak{Q}\sigma,\mathfrak{Q}\sigma,\mathfrak{P}\sigma,\lambda) * G_A(\mathfrak{Q}\sigma,\mathfrak{Q}\sigma,\mathfrak{P}\sigma,\lambda)$.
Then $\mathfrak{P}$ *and* $\mathfrak{Q}$ *have a one and only common fixed point.*

Proof. The confirmation follows on the same lines of Theorem (4.5) and by the meaning of $CLR_{\mathfrak{Q}}$ property. □

5. Conclusion

This article introduces a few mappings $(\mathfrak{P},\mathfrak{Q})$ on an AFMS, such as WC $(\mathfrak{J}_{\mathfrak{F}})$ and R-WC $(\mathfrak{J}_{\mathfrak{F}})$, and uses the aforementioned mapping in symmetric AFMS to determine the uniqueness of the common fixed point. Additionally, in symmetric AFMS, the common fixed point can be found by utilizing the property (E.A.) or $CLR_{\mathfrak{Q}}$ property.

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