

An Improved Residual Power Series Method for Radiative MHD CuO-Water Nanofluid Convection in a Semi-Porous Stretching Channel

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Abstract

This study considers radiative magnetohydrodynamic forced convection of a CuO–water nanofluid in a semi-porous channel with a stretching wall, with Joule heating included in the energy equation. By applying suitable similarity transformations, the governing continuity, momentum, and energy equations are reduced to a coupled system of nonlinear ordinary differential equations. The resulting boundary value problem is solved using the Improved Residual Power Series Method (IRPSM). The validity of the proposed approach is confirmed through comparison with available benchmark results, showing close agreement. The effects of the Reynolds number, Hartmann number, suction parameter, and nanoparticle shape on the velocity and temperature distributions are examined. The results indicate that the magnetic field suppresses the flow, suction significantly alters the thermal and momentum boundary layers, and platelet-shaped nanoparticles yield the highest local Nusselt number among the considered shapes. The study demonstrates that IRPSM is an effective and accurate semi-analytical tool for this class of nonlinear transport problems.

Keywords: Improved residual power series method; semi-porous channel; CuO-water nanofluid; thermal radiation; Joule heating; MHD forced convection.

1. Introduction

The unyielding drive towards reduction and increased power density in modern industrial and technological applications, from microelectronics and aerospace to biomedical engineering and renewable energy systems, has placed unprecedented demands on thermal management technologies. Conventional heat transfer fluids, such as water, oil, and ethylene glycol, are inherently limited by their relatively low thermal conductivity, creating a bottleneck for dissipating high heat fluxes efficiently [1, 2]. In response to this

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challenge, the concept of "nanofluids" engineered colloidal suspensions formed by dispersing nanometer-sized particles into a base fluid emerged as a revolutionary paradigm in heat transfer engineering[3]. The resulting suspensions exhibit remarkable enhancements in their effective thermal conductivity, attributed to mechanisms like Brownian motion-induced micro-convection, thermophoresis, and the formation of ordered liquid layers at the solid-liquid interface [4, 5]. Among these, copper oxide (CuO) nanoparticles have garnered significant attention due to their high thermal conductivity, cost-effectiveness, and unique properties like antimicrobial activity, making them a prime candidate for advanced cooling systems [6, 7].

While nanofluids offer a passive means of enhancing heat transfer, active control over the flow and thermal fields can be achieved through the application of external fields, leading to the study of magnetohydrodynamics (MHD). MHD investigates the behavior of electrically leading fluids, such as nanofluids, in the presence of a magnetic field. The interaction between the fluid's induced currents and the magnetic field gives rise to the Lorentz force, a body force that acts in opposition to the fluid motion [8]. This offers a powerful, non-intrusive mechanism to manipulate the flow field, suppress turbulence, and control heat transfer rates. The impact of MHD is profound; the Lorentz force acts as a velocity retardant, but its effect on heat transfer is nuanced. While it can reduce convective heat transfer by suppressing fluid motion, the associated Joule heating the generation of heat due to electrical resistance can counteract this effect by adding a volumetric heat source to the system [9]. This dual nature makes the analysis of MHD systems complex yet highly relevant for applications ranging from electromagnetic casting to MHD pumps [10].

In high temperature applications, such as those in aerospace and nuclear engineering, thermal radiation is no longer a negligible mode of heat transfer. Radiative heat transfer can be comparable to, or even dominate, conductive and convective heat transfer, fundamentally altering the temperature distribution within the system [11]. The Rosseland approximation is often employed to model this, simplifying the radiative flux into a diffusion-like term that depends on the temperature gradient, which couples with convection to create a highly nonlinear energy equation [12]. Furthermore, the geometry of the flow domain is critical. Semi-porous channels, characterized by walls that allow for fluid injection or suction (transpiration), model systems like filtration processes and transpiration cooling [13]. The addition of stretching or shrinking walls, as seen in polymer extrusion, further complicates the dynamics by introducing a moving boundary condition that directly affects the momentum and thermal boundary layers [14]. The combination of a semi-porous channel with stretching walls under the effect of a magnetic field and thermal radiation represents a multi-physics problem of significant complexity and industrial relevance.

The literature is replete with investigations into the various facets of this complex problem. Goudarzi and Ganji [15] pioneered the logical investigation of thermal radiation effects on nanofluid forced convection in a semi-porous channel, utilizing the Differential Transform Method (DTM) to solve the governing equations. Their work demonstrated that temperature gradients are significantly augmented with increases in the Reynolds number, suction parameter, and radiation parameter. They also highlighted the importance of nanoparticle shape, finding that platelet-shaped nanoparticles provided the ex-

treme Nusselt number [16]. Building on this, Jalili et al.[17] conducted a more comprehensive study, employing both the Akbari-Ganji Method (AGM) and the Homotopy Perturbation Method (HPM). Their research corroborated the earlier findings and provided further validation by comparing their analytical results with numerical solutions, emphasizing the significant impact of parameters like the Hartmann number (Ha) and the Eckert number (Ec) [18]. Beyond these specific studies, a vast body of work exists on related problems, exploring MHD nanofluid flow in cavities [19], over stretching sheets [20], and through porous media [21], with researchers often relying on sophisticated analytical techniques like the Homotopy Analysis Method (HAM) and Adomian Decomposition Method (ADM) to tackle the resulting highly nonlinear, coupled ordinary differential equations (ODEs)[22, 23, 24, 25, 26, 27, 28].

Despite these advances, a clear gap remains in the availability of analytical methods that are simultaneously robust, accurate, and computationally efficient for solving these complex, multi-parameter problems. The governing equations are characterized by strong nonlinearities arising from magnetic field terms, the radiation term, and the variable properties of the nanofluid. Existing methods, while valuable, often represent a tradeoff between accuracy, convergence speed, and implementation complexity. For instance, HPM and HAM can suffer from convergence issues or require the careful selection of an auxiliary parameter, while AGM can involve complex algebraic manipulations. To address these limitations, the Improved Residual Power Series Method (IRPSM) as a novel analytical technique [29, 30]. IRPSM is distinguished by its ability to provide rapidly convergent series solutions without the need for auxiliary parameters or complex linearization, offering a more direct and computationally efficient path to accurate solutions. This study, therefore, leverages the power of this new method by applying IRPSM to the present multi-physics problem, demonstrating its efficacy and providing a robust analytical solution.

The primary objective of this research is to perform a comprehensive analytical investigation into the forced convection of a CuO-water nanofluid in a semi-porous channel with a stretching wall, under the combined influence of a transverse magnetic field and thermal radiation. The specific aims are to formulate the governing equations, transform them into ODEs using similarity transformations, solve them using IRPSM, and validate the results against existing literature [15, 17]. A detailed parametric study will analyze the influence of key dimensionless parameters including the Reynolds number (R), Hartmann number (Ha), radiation parameter (R_d), Eckert number (Ec), and suction parameter (λ) on velocity and temperature profiles, as well as on the Nusselt number. This research will provide new physical insights into the flow and thermal behavior, contributing to the design and optimization of more efficient thermal management systems. The remainder of this paper is organized as follows: Section 2 presents the mathematical formulation, Section 3 describes the IRPSM and its application, Section 4 presents the results and discussion, and Section 5 provides the conclusions.

2. Mathematical Framework

This work considers a two-dimensional, steady, incompressible nanofluid moving through a channel bounded by semi-porous walls. As shown schematically in Fig. 1, the lower boundary is permeable and heated, whereas the upper boundary is non-permeable and kept at a lower temperature.

The nanofluid is treated as an electrically conducting suspension exposed to a uniform magnetic field applied normal to the main flow direction. To capture the relevant heat-transfer mechanisms, the energy balance includes thermal radiation as well as Joule (ohmic) dissipation generated by the induced electrical currents. The model is established from the standard conservation laws of mass, linear momentum, and energy (continuity and Navier–Stokes equations), and then extended by replacing the base-fluid material parameters with appropriate effective nanofluid properties. The required constitutive relations for these effective thermophysical properties are adopted following the approach reported in [15].

The flow is governed by the continuity equation, the momentum equations, and the energy equation [15]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.1)$$

$$\rho_{nf} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \sigma_{nf} B_0^2 u, \quad (2.2)$$

$$\rho_{nf} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_{nf} \frac{\partial^2 v}{\partial x^2}, \quad (2.3)$$

$$(\rho C_p)_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K_{nf} \frac{\partial^2 T}{\partial y^2} + \sigma_{nf} B_0^2 u^2 - \frac{\partial q_r}{\partial y}. \quad (2.4)$$

The effect of radiation is modeled by applying the Rosseland approximation, which relates the radiative heat flux to the temperature gradient. The radiative contribution depends on σ_e and β_R . Under this formulation, the heat flux q_r takes the form $q_r = -\frac{4\sigma_e}{3\beta_R} \frac{\partial T^4}{\partial y}$ [31].

A key simplifying assumption in this study is the linearization of the T^4 radiative term, which is justified by considering only small temperature differences within the fluid phase. Using a first-order Taylor expansion about T_c , we obtain $T^4 \cong 4T_c^3 T - 3T_c^4$. The model's fidelity is further enhanced by accurately characterizing the nanofluid effective properties. Accordingly, the effective density, effective heat capacity, and electrical conductivity are key material properties and are defined as follows [15]:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p, \quad (2.5)$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_p. \quad (2.6)$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3 \left(\frac{\sigma_p}{\sigma_f} - 1 \right) \phi}{\left(\frac{\sigma_p}{\sigma_f} + 2 \right) - \left(\frac{\sigma_p}{\sigma_f} - 1 \right) \phi}. \quad (2.7)$$

The characterization of the nanofluid requires the definition of its key effective properties. The effective density quantifies the overall mass of the nanofluid per unit volume, incorporating the contribution from the dispersed nanoparticle phase. The effective heat capacity measures the thermal energy necessary to raise the nanofluids temperature, a value derived from the respective heat capacities of the base fluid and the nanoparticles.

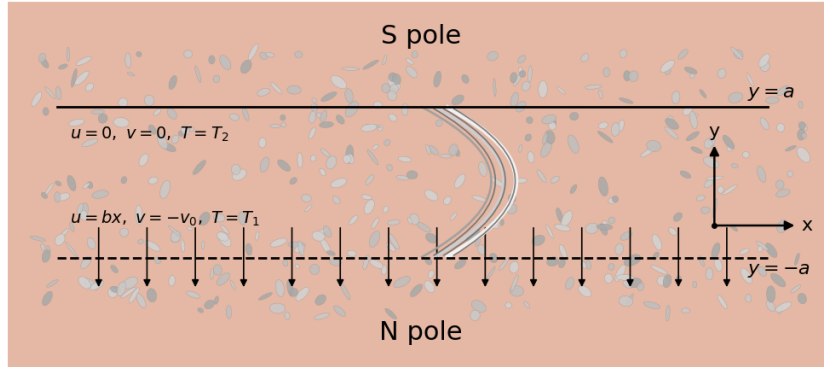


Figure 1: Schematic illustration of the semi-porous channel and boundary types.

Furthermore, the effective electrical conductivity denotes the fluids capacity for electrical conduction, a property sensitive to factors such as nanoparticles volumetric fraction, particle size, and the conductivity of the carrier fluid.

The accurate characterization of the nanofluids behavior under coupled physical phenomena such as magnetic field, thermal radiation, and joule heating is predicated on the precise definition of its effective thermophysical properties. This foundational step is critical for developing reliable models and predicting heat transfer characteristics across diverse application. In this study the nanofluids viscosity is specifically determined using the Koo-Kleinstreuer-Li (KKL) correlation [1, 32, 33, 34, 35]. This correlation is selected for its comprehensive approach, which estimates viscosity based on the theory that it is a function of nanoparticle characteristics, including size and morphology. By integrating the KKL correlation, this research achieves a more accurate representation of the fluids viscous behavior, thereby improving the understanding of the flow properties and the thermal performance of the system [34]:

$$\begin{aligned}\mu_{\text{eff}} &= \mu_{\text{static}} + \mu_{\text{Brownian}} = \mu_{\text{static}} + \frac{k_{\text{Brownian}}}{k_f} \times \frac{\mu_f}{P_{\text{rf}}}, \\ k_{\text{Brownian}} &= 5 \times 10^4 \phi \rho_f c_{p,f} \sqrt{\frac{k_b T}{\rho_p d_p}} g'(T, \phi, d_p), \\ g'(T, \phi, d_p) &= (a_1 + a_2 \ln(d_p) + a_3 \ln(\phi) \ln(d_p) + a_5 \ln(d_p)^2) \ln(T) \\ &\quad + (a_6 + a_7 \ln(d_p) + a_8 \ln(\phi) + a_9 \ln(\phi) \ln(d_p) + a_{10} \ln(d_p)^2).\end{aligned}\tag{2.8}$$

The coefficients and properties pertinent to the CuO-water nanofluid are detailed in Table 1. To represent the effective properties, the Maxwell model is applied alongside the Hamilton-Crosser model. Irregular particle morphology is accommodated in the Hamilton-Crosser formulation through the introduction of a shape factor, therefore the corresponding expression is [10, 36].

$$\frac{k_{\text{nf}}}{k_f} = \frac{k_p + (m+1)k_f - (m+1)\phi(k_f - k_p)}{k_p + (m+1)k_f + \phi(k_f - k_p)}.\tag{2.9}$$

In the preceding equation k_p and k_f denote the thermal conductivities of the nanoparticle material and the base fluid, respectively, while m represents the empirical shape factor, as detailed in [15]. Specific values for m correspond to different particle geometries: 3 for spherical, 3.7 for brick, 4.8 for cylindrical, and 5.7 for platelet shapes. For this investigation, copper oxide (CuO) nanoparticles are selected. This choice is motivated not only by their established efficacy in enhancing heat transfer metrics like the Nusselt number in water-based nanofluids but also by their multifunctional properties, which include antibacterial action against *E. coli* and inherent magnetic characteristics suitable for ferrofluid applications. This combination of features underscores the versatility of CuO as a high-performance material in advanced fluid systems.

Accordingly, the boundary conditions for the present model are given by:

$$\begin{aligned} u &= bx, \quad v = -v_0, \quad T = T_1 \quad \text{at} \quad y = -a, \\ u &= 0, \quad v = 0, \quad T = T_2 \quad \text{at} \quad y = a. \end{aligned} \quad (2.10)$$

The physical behavior of the channel walls is dictated by the parameter b s negative values signifies wall shrinking and a positive value signifies wall stretching. The mathematical formulation is simplified by applying a similarity transformation, which reduces the governing partial differential equations to a system of ordinary differential equations. This transformation simultaneously introduces key non-dimensional parameters, enabling a more generalized analysis of problem [15].

$$\eta = \frac{y}{a}, \quad u = bxf'(\eta), \quad v = -abf(\eta), \quad \theta = \frac{T - T_1}{T_1 - T_2}. \quad (2.11)$$

With the aid of the transformation in Eq. (11), Eqs. (12,13) are derived.

Material	ρ (kg/m ³)	c_p (J/(kg·K))	k (W/(m·K))	σ (Ω m) ⁻¹
CuO	6500	540	18	0.05
Pure water	997.1	4179	0.613	10^{-10}

Table 1: Thermo Physical Properties [15].

$$f^{iv} - Ha^2 \frac{A_5}{A_2} f'' - R \frac{A_1}{A_2} (f'f'' - ff''') = 0, \quad (2.12)$$

$$\left(1 + \frac{4}{3A_4} R_d\right) \theta'' + Pr \frac{A_3}{A_4} f \theta' + Ha^2 \frac{Ec}{R} \frac{Pr}{A_4} \frac{A_5}{A_4} f'^2 = 0. \quad (2.13)$$

The system of governing equations, specifically (2.8) and (2.9), is characterized by a set of non-dimensional parameters. These include the Reynolds number R , Eckert number Ec , Hartmann number Ha , the radiation parameter R_d , and Prandtl number Pr . Furthermore, the equations are defined by the constants A_1 through A_5 as detailed in [15].

$$\begin{aligned}
R &= \frac{a^2 b}{U_f}, \quad Ha = B_0 a \sqrt{\frac{\sigma_f}{\mu_f}}, \quad R_d = \frac{4\sigma_e T_c^3}{\beta_R k_f}, \\
Ec &= \frac{\rho_f (bx)^2}{(\rho C_p)_f (T_1 - T_2)}, \quad Pr = \frac{a^2 b (\rho C_p)_f}{k_f}, \\
A_1 &= \frac{\rho_{nf}}{\rho_f}, \quad A_2 = \frac{\mu_{nf}}{\mu_f}, \quad A_3 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, \quad A_4 = \frac{k_{nf}}{k_f}, \quad A_5 = \frac{\sigma_{nf}}{\sigma_f}.
\end{aligned} \tag{2.14}$$

In addition, the boundary conditions for the transformed ODEs can be written as follows:

$$\begin{aligned}
f'(-1) &= 1, & f'(1) &= 0, & \theta(-1) &= 1, \\
f(-1) &= \lambda = \frac{v_0}{ab}, & f(1) &= 0, & \theta(1) &= 0.
\end{aligned} \tag{2.15}$$

3. Methodology

3.1. Basic idea of IRPSM

The Improved Residual Power Series Method (IRPSM) is an extension of the Residual Power Series Method (RPSM) designed for boundary value problems. The key idea is to convert the boundary value problem into an initial-value-type formulation by introducing the missing initial quantities as unknown constants. A truncated power-series solution is then constructed, and the coefficients are determined recursively by imposing the vanishing of the residual and its successive derivatives. Finally, the unknown initial constants are obtained from the remaining boundary conditions. In this way, IRPSM avoids discretization, perturbation, and linearization, while still providing a rapidly convergent semi-analytical approximation.

The system under consideration is described by the following n th-order differential equation:

$$y^{(n)}(u) = g(u, y^{(m)}(u)), \quad 0 \leq u \leq a, \quad m = 0, 1, 2, \dots, n-1, \tag{3.1}$$

subject to the boundary conditions

$$y^{(m)}(u) = \beta_m, \quad m = 0, 1, 2, \dots, n-1, \tag{3.2}$$

and the initial conditions

$$y^{(m)}(0) = \alpha_m, \quad m = 0, 1, 2, \dots, n-1. \tag{3.3}$$

For Eqs. (3.2) and (3.3), y is the dependent unknown, while g and α_m are known quantities. Accordingly, a truncated power-series approximation of order k is introduced as

$$y(u) = \sum_{j=0}^k P_j u^j, \quad k = 0, 1, 2, \dots, n-1. \tag{3.4}$$

The quantities P_j denote the unknown initial parameters. For an n th-order boundary-value problem, these parameters are determined for $j = 0, 1, 2, \dots, n-1$. When $k = 0$, Eq. (3.4) takes the form

$$y(u) = \sum_{j=0}^0 P_j u^j. \quad (3.5)$$

Evaluating Eq. (3.5) at $u = 0$ and equating it to the initial condition leads to

$$P_0 = \alpha_0. \quad (3.6)$$

For the case $k = 1$, the first derivative of Eq. (3.4) evaluated at $u = 0$ is compared with the initial condition, resulting in

$$P_1 = \frac{\alpha_1}{1!}. \quad (3.7)$$

Proceeding similarly for $k = 2$, the resulting expression is

$$P_2 = \frac{\alpha_2}{2!}. \quad (3.8)$$

Proceeding to $k = n-1$, we compute the $(n-1)$ th derivative of Eq. (3.4) and set $u = 0$. Matching with the initial condition results in

$$P_{n-1} = \frac{1}{(n-1)!} \alpha_{n-1}. \quad (3.9)$$

In boundary-value formulations, unknown initial conditions are parameterized and later fixed by enforcing the boundary conditions. The rest of the constants P_j corresponding to $k = n, n+1, n+2, \dots$, can be computed using the following procedure. We first introduce a k th truncated series in the form

$$y(u) = y_{\text{initial}}(u) + \sum_{j=n}^k P_j u^j, \quad (3.10)$$

where $y_{\text{initial}}(u)$ is the k th truncated series. For $k = 0, 1, 2, \dots, n-1$, $y_{\text{initial}}(u)$ is given as

$$y_{\text{initial}}(u) = y^{(n-1)}(u) = P_0 + P_1 u + P_2 u^2 + \dots + P_{n-1} u^{n-1}. \quad (3.11)$$

Upon substituting P_j as defined in Eq. (3.11), the following expression is obtained:

$$y_{\text{initial}}(u) = \alpha_0 + \alpha_1 u + \frac{1}{2!} \alpha_2 u^2 + \dots + \frac{1}{(n-1)!} \alpha_{n-1} u^{n-1}. \quad (3.12)$$

Accordingly, Eq. (3.1) may be expressed in the form

$$y^{(n)} - g(u, y^{(m)}(u)) = 0. \quad (3.13)$$

Accordingly, combining Eq. (3.4) and Eq. (3.13) results in the k th-order residual function

$$\begin{aligned} \text{Res}^k(u) = & \sum_{j=n}^k j(j-1)(j-2) \dots (j-n+1) P_j u^{j-n} \\ & - g\left(u, \sum_{j=m}^k j(j-1)(j-2) \dots (j-m+1) P_j u^{j-m}\right). \end{aligned} \quad (3.14)$$

3.2. Residual of IRPSM

To find the residual of Eq. (3.1), we can write

$$\frac{d^{k-n}}{du^{k-n}} \text{Res}^k(0) = 0. \quad (3.15)$$

To obtain the n th approximate solution, set $k = n$ (where n is the order of the problem) in Eq. (3.15); we get

$$\begin{aligned} \frac{d^{k-n}}{du^{k-n}} \text{Res}^k(u_0) &= \frac{d^{k-n}}{du^{k-n}} \left(\sum_{j=n}^k j(j-1) \dots (j-n+1) P_j u^{j-n} \right) \\ &\quad - \frac{d^{k-n}}{du^{k-n}} \left\{ g \left(u, \sum_{j=m}^k j(j-1) \dots (j-m+1) P_j u^{j-m} \right) \right\}, \\ &\quad m = 0, 1, 2, \dots, n-1. \end{aligned} \quad (3.16)$$

Differentiating Eq. (3.16) $(k-n)$ times with respect to u and setting $u = u_0$, we get

$$\begin{aligned} \frac{d^{k-n}}{du^{k-n}} \text{Res}^k(u_0) &= \frac{d^{k-n}}{du^{k-n}} \left(\sum_{j=n}^k j(j-1)(j-2) \dots (j-n+1) P_j u^{j-n} \right) \\ &\quad - \frac{d^{k-n}}{du^{k-n}} \left\{ g \left(u, \sum_{j=m}^k j(j-1)(j-2) \dots (j-m+1) P_j u^{j-m} \right) \right\}, \quad m = 0, 1, 2, \dots, n-1. \end{aligned} \quad (3.17)$$

Accordingly, after computing P_n , the n th-order truncated series takes the form

$$y^n(u) = P_0 + P_1 u + P_2 u^2 + \dots + P_n u^n, \quad (3.18)$$

The process is continued iteratively until convergence to the required accuracy is obtained.

4. Results and discussion

This section presents the numerical and graphical results for radiative heat transfer in an MHD CuO-water nanofluid flowing through a semi-porous channel with a stretching wall. The energy equation includes the effect of Joule (ohmic) heating. By means of the similarity transformation, the governing partial differential equations are reduced to a coupled system of nonlinear ordinary differential equations, which is then solved using the Improved Residual Power Series Method (IRPSM). To verify the accuracy of the proposed method, the present results are compared with previously published numerical results, and a close agreement is observed, as illustrated in Figure. 2.

Figure 2 shows the variation of the velocity and temperature profiles for different values of the Reynolds number and Prandtl number for the fixed parameter values $R = 0.5$, $M = 1$, $\lambda = 0.5$, and $K = 0.5$. The present IRPSM solutions are compared with

the numerical results reported in the literature, and the observed agreement confirms the accuracy and reliability of the proposed method for the present nonlinear model. In addition, Table 2 presents the local Nusselt number for different nanoparticle shapes, namely cylinder, spherical, platelet, and brick, at $Pr = 6.2$ and several values of R . It is evident that the platelet-shaped nanoparticles yield the highest local Nusselt number among the considered cases, indicating superior heat-transfer performance.

Figure 3 illustrates the influence of the Reynolds number on the velocity and temperature distributions for $R_d = 0.5$, $Ha = 1$, $\lambda = 1$, $Ec = 0.5$, $\phi = 0.04$, and $Pr = 6.2$. As the Reynolds number increases, the temperature profile decreases, and the flow field is noticeably modified. The horizontal velocity near the lower wall decreases, whereas an opposite trend is observed near the upper wall. This behavior reflects the asymmetric transport mechanism induced by the stretching wall and the imposed channel boundary conditions.

Figure 4 presents the effect of the Hartmann number on the velocity and temperature profiles. With increasing Ha , the velocity decreases due to the strengthening of the Lorentz force, which resists the fluid motion. In contrast, the temperature profile increases as Ha increases. This rise in temperature is associated with enhanced Joule heating and weakened convective transport caused by magnetic damping, both of which contribute to thermal accumulation within the channel.

Figure 5 depicts the influence of the suction parameter on the velocity and temperature profiles. As λ increases, the temperature profile and the vertical velocity increase, while the horizontal velocity decreases. Moreover, the minimum point of the velocity profile shifts toward the lower wall. These results indicate that wall suction significantly alters the momentum and thermal boundary layers and can therefore be used as an effective control parameter for heat-transfer enhancement in the semi-porous channel configuration.

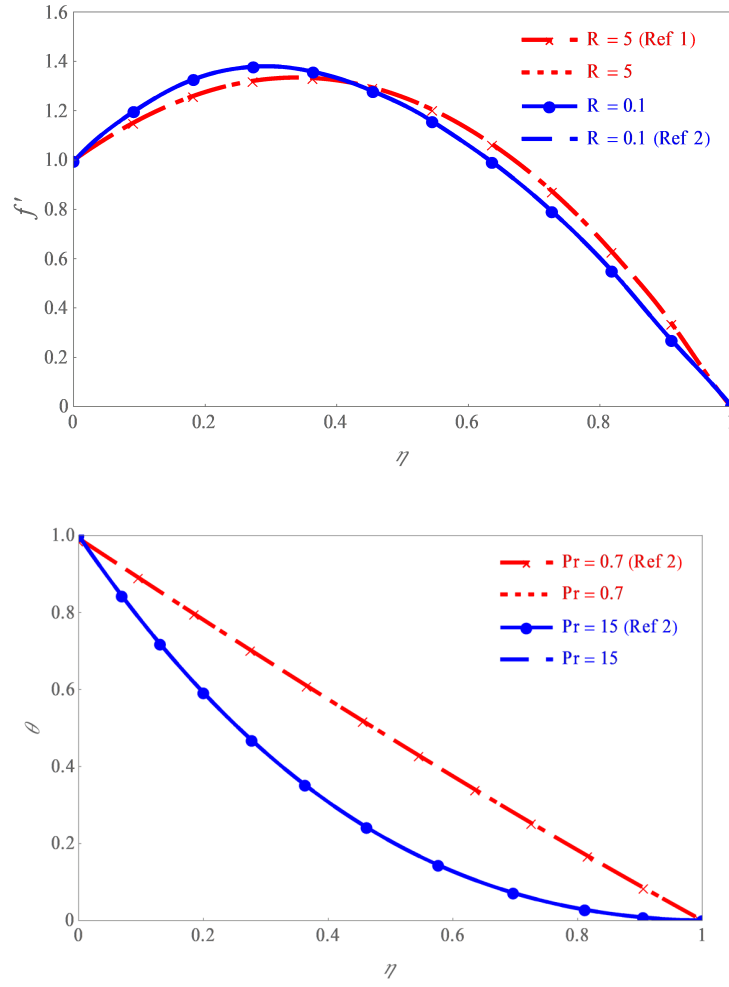
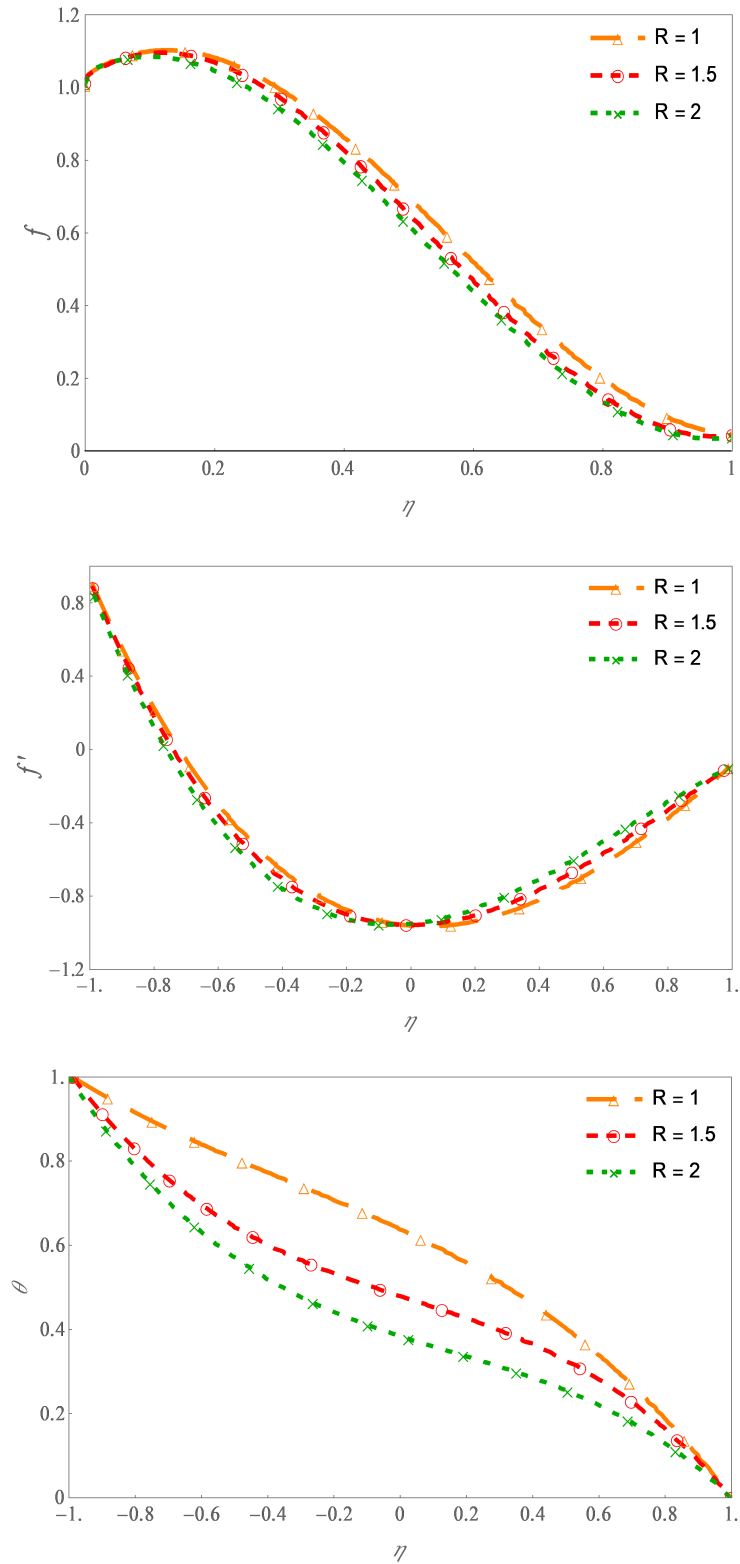
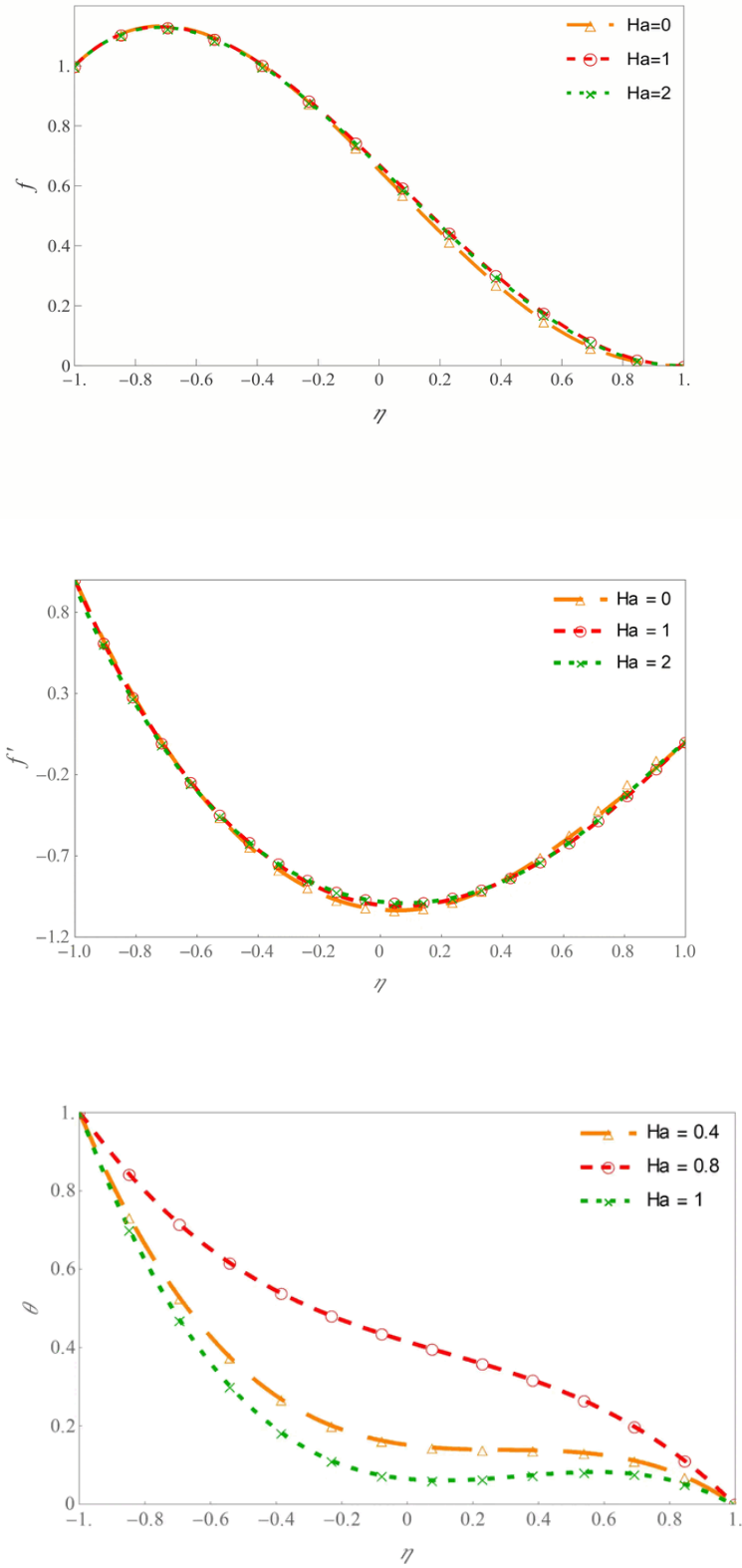
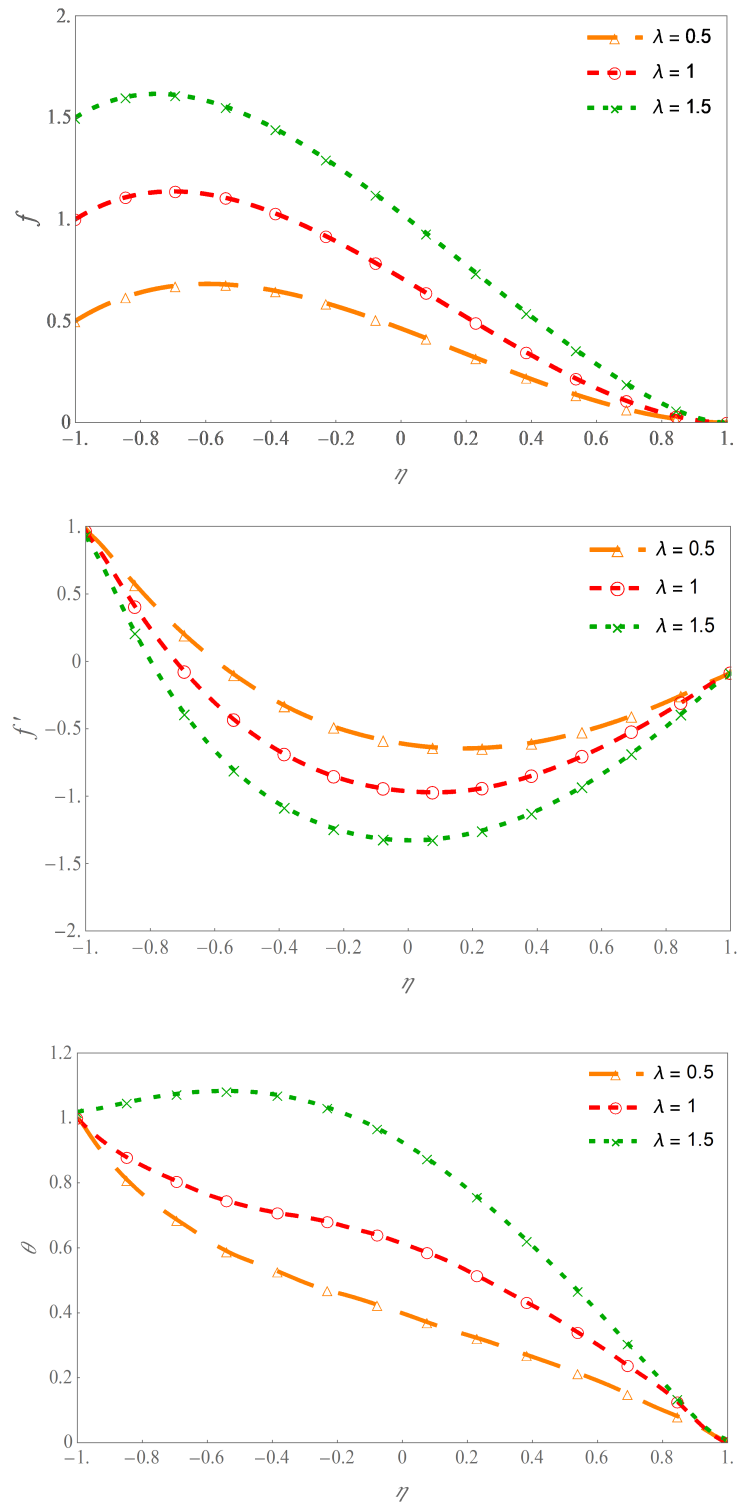


Figure 2: Investigation of how temperature and velocity profiles change with R and Pr when $R_d = 0.5$, $M = 1$, $\lambda = 0.5$, and $K_r = 0.5$, by comparing IRPSM results with the numerical method in [16, 15].

Figure 3: Investigation of how changes in R affect the temperature and velocity profiles via IRPSM.

Figure 4: Investigation of how changes in Ha affect the temperature and velocity profiles via IRPSM.

Figure 5: Investigation of how changes in λ effect the temperature and velocity profiles via IRPSM.

R	Cylinder	Spherical	Platelet	Brick
1	1.790366	1.735301	1.8151461	1.757658
1.5	3.247116	3.21957	3.259504	3.230759
2	3.905465	3.892636	3.91125	3.897842
2.5	4.240018	4.236621	4.241594	4.237987

Table 2: Nusselt number for various shapes of the nanoparticles when $Pr = 6.2$.

5. Conclusion

Following a comprehensive investigation of radiative magnetohydrodynamic CuO-water nanofluid flow and heat transfer in a semi-porous channel with a stretching wall, it can be concluded that the Improved Residual Power Series Method (IRPSM) provides an accurate, stable, and efficient framework for predicting the velocity and temperature fields of the present nonlinear system. The comparison with available benchmark results demonstrates close agreement, thereby confirming the reliability of the proposed method. The parametric analysis further reveals that the Reynolds number, Hartmann number, suction parameter, and nanoparticle shape significantly affect the flow and thermal characteristics within the channel. In particular, an increase in the Reynolds number reduces the temperature distribution and alters the velocity profiles, while a stronger magnetic field suppresses the fluid motion due to the Lorentz force and enhances the temperature field because of magnetic damping and Joule heating effects. Similarly, increasing the suction parameter modifies both the momentum and thermal boundary layers, leading to noticeable changes in the velocity structure and temperature distribution. Moreover, the analysis of nanoparticle shape shows that platelet-shaped nanoparticles produce the highest local Nusselt number among the considered geometries, indicating superior heat-transfer performance. Overall, the present study establishes IRPSM as a reliable semi-analytical tool for investigating coupled MHD nanofluid transport problems and highlights the important roles of magnetic control, wall suction, and nanoparticle geometry in improving thermal performance in semi-porous channel flows. Future work may extend the present formulation by considering variable thermophysical properties, particle-size effects, alternative nanoparticle materials, and more complex boundary conditions in order to further enhance the modeling of practical heat-transfer systems.

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