

Approximate solutions to malaria transmission models with seasonality and awareness: A comparative analysis of ADM and HPM

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Abstract

Mathematical modelling provides critical insights into malaria transmission dynamics and intervention efficacy. While standard numerical solvers provide quantitative solutions, semi-analytical methods yield closed-form series approximations that enhance qualitative and parametric understanding. This study comparatively investigates two prominent semi-analytical techniques, the Adomian Decomposition Method (ADM) and the Homotopy Perturbation Method (HPM), applied to a seasonally-forced malaria transmission model. The underlying formulation encompasses human behavioral awareness, a structured mosquito life cycle, and seasonal forcing, resulting in a twelve-dimensional coupled non-autonomous nonlinear system. Explicit recursive algorithms for ADM and HPM are derived and validated against fourth-fifth order Runge–Kutta numerical benchmarks using biologically realistic parameters. The comparative evaluation indicates compartment-dependent performance: HPM exhibits high precision for human epidemiological classes over short intervals, while ADM demonstrates superior stability across mosquito developmental stages and awareness dynamics over extended time periods. Numerical experiments further show that intensified awareness campaigns significantly curtail infection prevalence across both host and vector populations. The methods remain computationally tractable for 12D systems. Our findings therefore provide practical guidance for selecting semi-analytical schemes according to compartment and time-scale in high-dimensional epidemiological systems.

Keywords: Mathematical Modeling, Semi-Analytic Method, Non-linear Differential Equations, Epidemiology, Intervention Strategies, Seasonality.

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1. Introduction

Malaria is a parasitic disease caused by *Plasmodium* species and transmitted to humans by female *Anopheles* mosquitoes [39]. Following inoculation, sporozoites migrate through the circulatory system to hepatocytes for pre-erythrocytic development before invading red blood cells to initiate asexual cycles that produce recurring fever, chills, and

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severe anemia [16, 38]. Sustained transmission relies on reciprocal infectious interactions between the human host reservoir and mosquito vectors, creating intricate transmission dynamics governed by ecological, environmental, and behavioral determinants.

In this context, mathematical modeling has established itself as an indispensable tool for unraveling transmission mechanisms, characterizing epidemic thresholds, and evaluating control strategies. Compartmental formulations based on the Susceptible–Exposed–Infectious–Recovered–Susceptible (SEIRS) framework have proven particularly effective in capturing the core mechanisms of disease spread. In recent years, mathematical epidemiology has progressed substantially by moving beyond classical homogeneous formulations to incorporate greater biological and environmental realism [1, 3, 6, 9, 13, 15, 20, 26, 27, 32, 33]. First, modern frameworks explicitly capture the complete entomological life cycle of vectors by stratifying mosquito populations into aquatic immature stages (eggs, larvae, pupae) and terrestrial adults [15, 17, 25, 32]. This stage-structured formulation captures critical biological bottlenecks, including density-dependent larval competition, environmental carrying capacities, and stage-specific mortality. Second, because vector abundance, biting rates, and parasite sporogony are heavily regulated by meteorological variables such as precipitation, ambient temperature, and humidity, recent models explicitly embed environmental seasonality through periodic forcing functions and climate-driven transmission parameters [6, 14, 30, 33]. Third, recognizing that disease transmission is tightly linked to human host behavior, contemporary studies integrate the dynamic impact of public awareness campaigns [7, 13, 19, 20, 27, 26, 28, 29]. Through mass media broadcasts, social communication, and community outreach, awareness programs encourage behavioral adaptations, such as the consistent utilization of insecticide-treated bed nets (ITNs), indoor residual spraying, and prompt healthcare-seeking behavior. Mathematically, this is captured by dividing the host population into unaware and aware cohorts, introducing awareness-driven transition rates, and applying reduced transmission coefficients to protected individuals.

A prominent example of such an advanced formulation is the seasonally forced malaria transmission model developed by Ochieng [33]. This model couples a human host population stratified by awareness levels with a structured mosquito life cycle encompassing aquatic developmental stages (eggs, larvae, pupae) and adult vectors, together with a dynamic awareness campaign variable. The author parameterized the system with empirical incidence data from Kenya, derived a basic reproduction number $R_0 = 2.4537$, and evaluated future disease patterns using numerical integration routines, specifically the fourth-fifth order Runge–Kutta technique. However, while numerical methods provide reliable trajectories for specific parameter sets (see [12, 21, 37]), they do not yield explicit functional series representations and can become computationally demanding during extensive sensitivity analyses and structural parameter sweeps. This limitation underscores the value of powerful semi-analytical methods, which generate rapidly convergent series solutions that provide closed-form analytical approximations and reveal direct insights into the functional relationships governing model compartments.

Among semi-analytical techniques, the Adomian Decomposition Method (ADM), introduced by Adomian [5], offers an effective platform for solving nonlinear functional and differential equations without requiring linearization, perturbation, or discretization. Its central innovation is the systematic decomposition of nonlinear operators into series

of specially generated Adomian polynomials. In epidemiological modeling, Abioye et al. [2] demonstrated the efficacy of ADM for a seven-compartment malaria model, achieving close agreement with numerical solutions. Extending this framework, Adebisi et al. [4] applied ADM to an infectious disease transmission model, showing that the resulting series approximations provide high numerical accuracy and rapid convergence. More recently, Yunus and Olayiwola [40] utilized decomposition methods to analyze the co-dynamics of vector-borne and viral transmission models involving malaria, confirming the ability of Adomian-based series solutions to capture complex multi-compartment interactions.

Concurrently, the Homotopy Perturbation Method (HPM), formulated by He [22, 23], synthesizes classical perturbation techniques with topological homotopy theory. A principal advantage of HPM is its freedom from small perturbation parameters, making it particularly well-suited for biological and epidemiological systems where natural perturbation parameters do not exist. In malaria modeling, Senthamarai and Balamuralitharan [36] successfully employed HPM to solve an SIRS–SI malaria model with treatment interventions, obtaining comprehensive analytical approximations. Recent investigations have further demonstrated the robustness of HPM in epidemiology: Geethamalini et al. [18] applied HPM to analyze a SIRC epidemic framework with high numerical fidelity; Rafiullah et al. [34] investigated a six-compartment nonlinear epidemic model via HPM, highlighting its efficacy for parameter control and sensitivity analysis; and Alshomrani [11] developed canonical HPM schemes for nonlinear epidemic systems, demonstrating superior analytical accuracy over traditional numerical integration routines.

The complementary virtues of ADM and HPM have motivated several recent investigations that deploy or comparatively evaluate both techniques across infectious disease models. For example, Alkahtani [8] performed an analytical investigation of a vaccinated SARS-CoV-2 epidemic model, constructing series-type solutions using both Laplace–Adomian decomposition and homotopy perturbation methods to analyze disease progression. Similarly, Mahreen et al. [31] obtained approximate analytical solutions for a nonlinear MERS-CoV transmission model by implementing the Adomian decomposition method and directly comparing the resulting series against the Homotopy Perturbation Method. In a related study, Riabi et al. [35] solved a fractional SIR epidemic model using HPM coupled with integral transforms and benchmarked the behavior of the infinite series solutions against ADM and Laplace–Adomian schemes. Furthermore, Alshammari et al. [10] developed homotopy perturbation transform and decomposition methodologies to solve a nonlinear SEIR measles epidemic framework, demonstrating the high precision and analytical flexibility of these semi-analytical schemes.

Despite these notable individual and comparative successes [2, 4, 8, 10, 11, 18, 31, 34, 35, 36, 40], existing applications of ADM and HPM in mathematical biology have largely been confined to lower-dimensional models (typically 3 to 7 state variables) without seasonal forcing or multi-stage vector development. Furthermore, direct comparative evaluations assessing the relative convergence rates, truncation errors, and compartment-specific precision of ADM versus HPM on high-dimensional nonlinear systems remain scarce. The seasonally forced malaria model formulated in [33] represents a rigorous and demanding benchmark, comprising twelve coupled nonlinear ordinary differential equations, logistic aquatic vector growth, time-periodic transmission rates, and nonlinear awareness-dependent terms. Therefore, the novel contribution of this work is the first application

and comparative analysis of the Adomian Decomposition Method and the Homotopy Perturbation Method to obtain semi-analytical solutions for the sophisticated, non-linear, and seasonally-forced malaria transmission model proposed by [33].

This paper therefore aims at deriving the recursive schemes for the 12-dimensional system for both ADM and HPM, explicitly computing the Adomian polynomials for the non-linear terms and presenting the first few terms of the series solutions. The semi-analytical results will then be validated against the numerical benchmark using biologically feasible parameters' values. Furthermore, we will compare the convergence efficiency and computational accuracy of ADM versus HPM for this specific biological system.

The remainder of this paper is however structured as follows: Section 2 briefly outlines the malaria model. Section 3 details the methodologies of the Adomian Decomposition and Homotopy Perturbation Methods. Sections 4 and 5 respectively present the application of the Adomian Decomposition Method (ADM) and Homotopy Perturbation Method (HPM) to the considered Malaria Model. Section 6 showcases numerical simulations, results, and a comparative discussion. The last section, Section 7 concludes the paper.

2. Mathematical Model Formulation

This study is based on the sophisticated deterministic compartmental model for malaria transmission developed in [33]. This model is notable for its integration of human behavioral awareness, a structured mosquito population, and seasonal forcing, providing a high degree of biological realism. The following section outlines the key components, assumptions, and governing equations of this model.

2.1. Model Structure and Compartments

The model stratifies the population into two interacting groups: the human host population and the mosquito vector population, with an additional state variable for population-level awareness.

2.1.1. Human Population

The human population is divided into five compartments:

- **Susceptible Unaware Humans (S_{hu}):** Individuals who are susceptible to malaria infection and are not aware of preventive measures.
- **Susceptible Aware Humans (S_{ha}):** Individuals who are susceptible but are aware of the disease and are taking preventive measures (e.g., bed nets, insecticides).
- **Exposed Humans (E_h):** Individuals who have been infected but are in the latent period and not yet infectious.
- **Infectious Humans (I_h):** Individuals who have developed clinical symptoms and can transmit the parasite to mosquitoes.
- **Recovered Humans (R_h):** Individuals who have recovered from the infection and possess temporary immunity.

The total human population is given by:

$$N_h(t) = S_{hu} + S_{ha} + E_h + I_h + R_h \quad (2.1)$$

2.1.2. Mosquito Population

The mosquito population is structured into six compartments to account for its life cycle:

- **Immature Stages:** Eggs (E), Larvae (L), and Pupae (P).
- **Mature Stages:**
 - **Susceptible Mosquitoes (S_m):** Adult mosquitoes that are not carrying the parasite.
 - **Exposed Mosquitoes (E_m):** Mosquitoes that have ingested the parasite but are not yet infectious.
 - **Infectious Mosquitoes (I_m):** Mosquitoes that are capable of transmitting the parasite to humans.

The total mature mosquito population is given by:

$$N_m(t) = S_m + E_m + I_m \quad (2.2)$$

2.1.3. Awareness Campaign ($A(t)$)

A distinct compartment, ($A(t)$), represents the level of public awareness driven by media campaigns. This awareness impacts the model by:

1. Transitioning individuals from S_{hu} to S_{ha} .
2. Reducing the force of infection for aware individuals S_{ha} .
3. Increasing the recovery rate of infectious humans I_h .
4. Increasing the mortality of mosquito variants (eggs, larvae, pupae, adults) through insecticide use.

2.2. Model Assumptions

The model is derived under the following key assumptions:

1. The growth of the immature mosquito stages (eggs, larvae, pupae) is governed by a logistic growth function.
2. The force of infection for aware susceptible humans S_{ha} is reduced by a factor of $(1 + A)$.
3. Awareness campaigns lead to the elimination of mosquito variants at a rate ΥA .
4. Recovered humans lose immunity at a rate φ and re-enter the susceptible unaware compartment.
5. Disease-induced mortality is considered for the human population.
6. The transmission rates (β_1, β_2) are subject to seasonal forcing.

2.3. Governing Equations

The dynamics of the system are governed by the following system of twelve coupled, nonlinear ordinary differential equations [33]:

2.3.1. Human Population Dynamics:

$$\frac{dS_{hu}}{dt} = b_h - \beta_2 S_{hu} I_m + (1 - \xi) \varphi R_h - \alpha_1 A S_{hu} + \frac{\alpha_2 S_{ha}}{1 + A} + (1 - q) \lambda E_h - \mu S_{hu} \quad (2.3)$$

$$\frac{dS_{ha}}{dt} = -\frac{\beta_1 S_{ha} I_m}{1 + A} + \xi \varphi R_h + \alpha_1 A S_{hu} - \frac{\alpha_2 S_{ha}}{1 + A} + q \lambda E_h - \mu S_{ha} \quad (2.4)$$

$$\frac{dE_h}{dt} = \frac{\beta_1 S_{ha} I_m}{1 + A} + \beta_2 S_{hu} I_m - (\mu + \lambda + \rho) E_h \quad (2.5)$$

$$\frac{dI_h}{dt} = \rho E_h - (\mu + \sigma + rA) I_h \quad (2.6)$$

$$\frac{dR_h}{dt} = rA I_h - (\mu + \varphi) R_h \quad (2.7)$$

2.3.2. Mosquito Population Dynamics:

$$\frac{dE}{dt} = b_m \left(1 - \frac{E}{K_E}\right) N_m - (\tau + d_E + \gamma A) E \quad (2.8)$$

$$\frac{dL}{dt} = \tau \left(1 - \frac{L}{K_L}\right) E - (v + d_L + \gamma A) L \quad (2.9)$$

$$\frac{dP}{dt} = v \left(1 - \frac{P}{K_P}\right) L - (\varepsilon + d_P + \gamma A) P \quad (2.10)$$

$$\frac{dS_m}{dt} = \varepsilon P - \kappa_1 S_m I_h - \kappa_2 S_m R_h - (\eta + \gamma A) S_m \quad (2.11)$$

$$\frac{dE_m}{dt} = \kappa_1 S_m I_h + \kappa_2 S_m R_h - (\chi + \eta + \gamma A) E_m \quad (2.12)$$

$$\frac{dI_m}{dt} = \chi E_m - (\eta + \gamma A) I_m \quad (2.13)$$

2.3.3. Awareness Campaign Dynamics:

$$\frac{dA}{dt} = \omega I_m + \delta I_h - \theta A \quad (2.14)$$

Where the initial conditions are all non-negative: $S_{hu}(0) > 0$, $S_{ha}(0) > 0$, ..., $A(0) > 0$.

2.4. Seasonality and Parameter Fitting

The model incorporates seasonal variation by multiplying the transmission rates (β_1 , β_2) by a seasonality factor:

$$s(t) = \left(1 + \cos\left(\frac{\pi}{15}t\right)\right). \quad (2.15)$$

The model parameters are estimated by fitting the system to real malaria incidence data from Kenya (2000–2021) using a least-squares curve-fitting algorithm, yielding a Mean Absolute Error (MAE) of 2.6463, which indicates a close alignment with observed data [detail can be found in [33]].

Table 1: Description of model variables.

Variable	Meaning
$S_{hu}(t)$	Population of Susceptible Unaware Humans at time t
$S_{ha}(t)$	Population of Susceptible Aware Humans at time t
$S_m(t)$	Population of Susceptible mature mosquitoes at time t
$E_h(t)$	Population of Exposed/Latent Humans at time t
$E_m(t)$	Population of Exposed/Latent mature mosquitoes at time t
$I_h(t)$	Population of Infectious Humans at time t
$I_m(t)$	Population of Infectious mature mosquitoes at time t
$R_h(t)$	Population of Recovered Humans at time t
$E(t)$	Population of Mosquito eggs in the environment at time t
$L(t)$	Population of Larvae in the environment at time t
$P(t)$	Population of Pupae in the environment at time t
$A(t)$	Level of awareness due to media campaign at time t

This comprehensive model, with its nuanced structure and data-driven parameterization, serves as a robust foundation for our subsequent analysis using the Adomian Decomposition and Homotopy Perturbation Methods.

3. Methodology

This section presents the fundamental algorithms of the two semi-analytical methods employed in this study: the Adomian Decomposition Method (ADM) and the Homotopy Perturbation Method (HPM). The general principles are outlined below.

3.1. Adomian Decomposition Method (ADM)

The Adomian Decomposition Method is a powerful technique for solving nonlinear functional equations of various kinds, including ordinary and partial differential equations [5]. The core of the method lies in decomposing the solution into an infinite series and elegantly handling nonlinear terms through the use of Adomian polynomials.

Consider a general nonlinear differential equation expressed in canonical form:

$$Ly + Ry + Ny = g(t) \quad (3.1)$$

where:

- L is the highest-order linear differential operator (invertible),
- R is the remainder of the linear operator,
- Ny represents the nonlinear terms,
- $g(t)$ is a known analytic function.

Applying the inverse operator L^{-1} to both sides yields:

$$y = \phi(t) + L^{-1}(g(t)) - L^{-1}(Ry) - L^{-1}(Ny) \quad (3.2)$$

Table 2: Description of model parameters.

Parameter	Meaning
b_h	Growth rate of susceptible unaware human population (either by birth or immigration)
b_m	Rate at which female anopheles mosquitoes lay eggs
α_1	Rate of awareness by media campaign
α_2	Rate at which aware people become unaware due to negligence
β_1	Rate of disease transmission due to infected mosquito bites
β_2	Rate of disease transmission due to infected mosquito bites
μ	Natural human death rate
σ	Death rate of human due to mosquito bite
λ	Progression rate from E_h to Susceptible due to robust immune system
r	Rate of recovery of the infectious human population due to medication
ρ	Progression rate from E_h to I_h
φ	Per capita rate of loss of immunity in humans
q	Probability of progression from E_h back to S_{ha}
ξ	Probability of progression from R_h back to S_{ha}
θ	Rate at which A declines due to fading of memory
δ	Rate at which A increases due to local campaigns
η	Natural death rate of mature mosquitoes in the environment
d_E	Natural death rate of mosquito eggs in the environment
d_L	Natural death rate of larvae in the environment
d_P	Natural death rate of pupae in the environment
τ	Progression rate from E stage to L stage
υ	Progression rate from L stage to P stage
ε	Progression rate from P stage to susceptible mature mosquito
κ_1	Infection rate of susceptible mosquitoes due to biting infectious humans
κ_2	Infection rate of susceptible mosquitoes due to biting recovered humans
χ	Progression rate from E_m to I_m
γ	Efficacy of insecticide due to the campaign awareness
K_E	Breeder sites occupied by mosquito eggs
K_L	Breeder sites occupied by larvae
ω	Rate at which A rises due to the campaign through global sources

where $\phi(t)$ arises from the initial conditions.

The ADM proposes that the solution $y(t)$ can be decomposed into an infinite series:

$$y(t) = \sum_{n=0}^{\infty} y_n(t) \quad (3.3)$$

The true genius of the method is the decomposition of the nonlinear term Ny into a series of specially generated polynomials:

$$Ny = \sum_{n=0}^{\infty} A_n \quad (3.4)$$

where A_n are the Adomian polynomials, which depend on the solution components $y_0, y_1, \dots, y_n$. These polynomials can be generated using the formula:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N \left(\sum_{k=0}^n \lambda^k y_k \right) \right]_{\lambda=0}, \quad \text{for } n = 0, 1, 2, 3, \dots \quad (3.5)$$

The first few Adomian polynomials for a general nonlinear term Ny are:

$$A_0 = N(y_0) \quad (3.6)$$

$$A_1 = y_1 N'(y_0) \quad (3.7)$$

$$A_2 = y_2 N'(y_0) + \frac{1}{2!} y_1^2 N''(y_0) \quad (3.8)$$

$$A_3 = y_3 N'(y_0) + y_1 y_2 N''(y_0) + \frac{1}{3!} y_1^3 N'''(y_0) \quad (3.9)$$

The recursive relationship for the solution components is then given by:

$$y_0 = \phi(t) + L^{-1}(g(t)) \quad (3.10)$$

$$y_{n+1} = -L^{-1}(Ry_n) - L^{-1}(A_n), \quad n \geq 0 \quad (3.11)$$

The n -term approximation of the solution can be written as $\phi_n(t) = \sum_{k=0}^{n-1} y_k(t)$, which converges to the exact solution as $n \rightarrow \infty$:

$$\lim_{n \rightarrow \infty} \phi_n(t) = y(t) \quad (3.12)$$

3.1.1. Convergence of the Adomian Decomposition Method (ADM)

The convergence of ADM has been extensively studied, and its theoretical foundation is more established for a specific class of problems. A common and powerful approach is to study ADM within the framework of Banach's Fixed Point Theorem. The standard convergence theorem for ADM can be stated as follows:

Consider the nonlinear equation $u = L(u) + N(u) + g$, reformulated for ADM. If L and N are contractive mappings (or can be bounded by a contractive operator) in a Banach space, then the series solution generated by ADM converges strongly to the unique solution of the equation. Under the conditions of the above theorem, the convergence of ADM is typically linear.

The error bound between the N th partial sum S_N and the exact solution u satisfies an inequality of the form:

$$\|u - S_N\| \leq k \cdot \|u - S_{N-1}\| \quad (3.13)$$

for some constant $0 < k < 1$. This characterizes linear convergence.

In practice, this means that each new term u_n you add to the series reduces the error by a roughly constant factor. The method is reliable and predictable for a wide range of problems, especially those where the nonlinearity is well-behaved.

The calculation of the Adomian polynomials is exact for the given nonlinearity. Therefore, the convergence rate is not hampered by approximations in the nonlinear terms themselves but by the fundamental recursive structure of the method.

3.2. Homotopy Perturbation Method (HPM)

The Homotopy Perturbation Method elegantly merges classical perturbation and homotopy techniques from topology to solve nonlinear problems [22, 23]. Its significant advantage is that it does not depend on the existence of a small parameter in the equation.

Consider a general nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega \quad (3.14)$$

with boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \quad (3.15)$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function, and Γ is the boundary of the domain Ω .

The operator A can be decomposed into linear (L) and nonlinear (N) parts. Thus, the equation can be rewritten as:

$$L(u) + N(u) - f(r) = 0 \quad (3.16)$$

By the homotopy technique, we construct a homotopy $v(r, p) : \Omega \times [0, 1] \rightarrow \mathbb{R}$ that satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \quad (3.17)$$

where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation satisfying the boundary conditions.

Alternatively, this can be written as:

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad (3.18)$$

From this construction, we have:

$$H(v, 0) = L(v) - L(u_0) = 0 \quad (3.19)$$

$$H(v, 1) = A(v) - f(r) = 0 \quad (3.20)$$

As p varies from 0 to 1, the solution $v(r, p)$ deforms from the initial approximation $u_0(r)$ to the actual solution $u(r)$ of the original equation. The embedding parameter p is treated as a "small parameter", and the solution v is assumed as a power series in p :

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots \quad (3.21)$$

Setting $p = 1$ results in the approximate solution of the original equation:

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + v_3 + \cdots \quad (3.22)$$

The components $v_0, v_1, v_2, v_3, \dots$ are determined by collecting terms with like powers of p after substituting the series into the homotopy equation.

3.2.1. Convergence of the Homotopy Perturbation Method (HPM)

The convergence analysis for HPM is often more varied and can be problem-dependent because of the freedom in choosing the initial guess and the auxiliary linear operator. Several convergence theorems exist for HPM. A common one, again leveraging fixed-point theory, states:

If the homotopy series $H(v, p)$ constructed by HPM converges uniformly as $p \rightarrow 1$, and the linear operator L in the homotopy is invertible, then the resulting series $\sum u_n$ is a solution to the original nonlinear problem.

The convergence rate of HPM is not fixed and can be significantly influenced by two key factors:

- **The Auxiliary Linear Operator (L):** A clever choice of L can dramatically accelerate convergence. A poor choice can lead to slow convergence or even divergence. This is both a weakness and a strength, as it provides a "convergence control parameter."
- **The Initial Guess (u_0):** A good initial guess close to the true solution will generally lead to faster convergence.

In its standard form, without any optimization of L or u_0 , HPM often exhibits convergence similar to a geometric series, which is also linear convergence. However, it is frequently observed that for many problems, HPM can converge faster than the standard ADM for the same number of terms. This is often attributed to the "homotopy" aspect, which can embed a better asymptotic structure for the problem than the straightforward decomposition in ADM.

HPM offers more flexibility. By introducing an auxiliary parameter (like the h -parameter in the Homotopy Analysis Method (HAM), a more advanced variant), one can exert explicit control over the convergence region and rate, sometimes achieving super-linear or even quadratic convergence for ideal choices.

4. Application of the Adomian Decomposition Method

Let $L = \frac{d(\cdot)}{dt}$ and the inverse operator L^{-1} is defined as $L^{-1} = \int_0^t (\cdot) dy$. Also, let $L^{-1}(L(y)) = y - y(0)$.

Multiplying each equation in the system of the malaria model by L^{-1} , we get:

For Human Population Dynamics:

$$\begin{aligned}
S_{hu}(t) - S_{hu}(0) &= L^{-1}[b_h] - \beta_2 L^{-1}[S_{hu}I_m] + (1 - \xi)\varphi L^{-1}[R_h] \\
&\quad - \alpha_1 L^{-1}[AS_{hu}] + \alpha_2 L^{-1}\left[\frac{S_{ha}}{1+A}\right] + (1 - q)\lambda L^{-1}[E_h] - \mu L^{-1}[S_{hu}] \\
S_{ha}(t) - S_{ha}(0) &= -\beta_1 L^{-1}\left[\frac{S_{ha}I_m}{1+A}\right] + \xi\varphi L^{-1}[R_h] + \alpha_1 L^{-1}[AS_{hu}] \\
&\quad - \alpha_2 L^{-1}\left[\frac{S_{ha}}{1+A}\right] + q\lambda L^{-1}[E_h] - \mu L^{-1}[S_{ha}] \\
E_h(t) - E_h(0) &= \beta_1 L^{-1}\left[\frac{S_{ha}I_m}{1+A}\right] + \beta_2 L^{-1}[S_{hu}I_m] - (\mu + \lambda + \rho)L^{-1}[E_h] \\
I_h(t) - I_h(0) &= \rho L^{-1}[E_h] - (\mu + \sigma)L^{-1}[I_h] - rL^{-1}[AI_h] \\
R_h(t) - R_h(0) &= rL^{-1}[AI_h] - (\mu + \varphi)L^{-1}[R_h]
\end{aligned}$$

For Mosquito Population Dynamics:

$$\begin{aligned}
E(t) - E(0) &= b_m L^{-1}[N_m] - \frac{b_m}{K_E} L^{-1}[ES_m] - \frac{b_m}{K_E} L^{-1}[EE_m] \\
&\quad - \frac{b_m}{K_E} L^{-1}[EI_m] - (\tau + d_E)L^{-1}[E] - \gamma L^{-1}[AE] \\
L(t) - L(0) &= \tau L^{-1}[E] - \frac{\tau}{K_L} L^{-1}[LE] - (v + d_L)L^{-1}[L] - \gamma L^{-1}[AL] \\
P(t) - P(0) &= v L^{-1}[L] - \frac{v}{K_P} L^{-1}[PL] - (\varepsilon + d_P)L^{-1}[P] - \gamma L^{-1}[AP] \\
S_m(t) - S_m(0) &= \varepsilon L^{-1}[P] - \kappa_1 L^{-1}[S_m I_h] - \kappa_2 L^{-1}[S_m R_h] - \eta L^{-1}[S_m] - \gamma L^{-1}[AS_m] \\
E_m(t) - E_m(0) &= \kappa_1 L^{-1}[S_m I_h] + \kappa_2 L^{-1}[S_m R_h] - (\chi + \eta)L^{-1}[E_m] - \gamma L^{-1}[AE_m] \\
I_m(t) - I_m(0) &= \chi L^{-1}[E_m] - \eta L^{-1}[I_m] - \gamma L^{-1}[AI_m]
\end{aligned}$$

For Awareness Campaign Dynamics:

$$A(t) - A(0) = \omega L^{-1}[I_m] + \delta L^{-1}[I_h] - \theta L^{-1}[A]$$

Rearranging each of the last set of equations gives

For Human Population Dynamics:

$$\begin{aligned} S_{hu}(t) = & S_{hu}(0) + b_h t - \beta_2 \int_0^t \left(\sum_{n=0}^{\infty} S_n \right) dy + (1 - \xi) \varphi \int_0^t \left(\sum_{n=0}^{\infty} R_{hn} \right) dy \\ & - \alpha_1 \int_0^t \left(\sum_{n=0}^{\infty} B_n \right) dy + \alpha_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{6n} \right) dy + (1 - q) \lambda \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy \\ & - \mu \int_0^t \left(\sum_{n=0}^{\infty} S_{hu(n)} \right) dy \end{aligned}$$

$$\begin{aligned} S_{ha}(t) = & S_{ha}(0) - \beta_1 \int_0^t \left(\sum_{n=0}^{\infty} C_n \right) dy + \xi \varphi \int_0^t \left(\sum_{n=0}^{\infty} R_{hn} \right) dy + \alpha_1 \int_0^t \left(\sum_{n=0}^{\infty} B_n \right) dy \\ & - \alpha_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{6n} \right) dy + q \lambda \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy - \mu \int_0^t \left(\sum_{n=0}^{\infty} S_{ha(n)} \right) dy \end{aligned}$$

$$E_h(t) = E_h(0) + \beta_1 \int_0^t \left(\sum_{n=0}^{\infty} C_n \right) dy + \beta_2 \int_0^t \left(\sum_{n=0}^{\infty} S_n \right) dy - (\mu + \lambda + \rho) \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy$$

$$I_h(t) = I_h(0) + \rho \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy - (\mu + \sigma) \int_0^t \left(\sum_{n=0}^{\infty} I_{hn} \right) dy - r \int_0^t \left(\sum_{n=0}^{\infty} D_n \right) dy$$

$$R_h(t) = R_h(0) + r \int_0^t \left(\sum_{n=0}^{\infty} D_n \right) dy - (\mu + \varphi) \int_0^t \left(\sum_{n=0}^{\infty} R_{hn} \right) dy$$

For Mosquito Population Dynamics:

$$\begin{aligned} E(t) = & E(0) + b_m \int_0^t \left(\sum_{n=0}^{\infty} (S_{mn} + E_{mn} + I_{mn}) \right) dy - \frac{b_m}{K_E} \int_0^t \left(\sum_{n=0}^{\infty} F_n \right) dy \\ & - \frac{b_m}{K_E} \int_0^t \left(\sum_{n=0}^{\infty} G_n \right) dy - \frac{b_m}{K_E} \int_0^t \left(\sum_{n=0}^{\infty} H_n \right) dy - (\tau + d_E) \int_0^t \left(\sum_{n=0}^{\infty} E_n \right) dy \\ & - \gamma \int_0^t \left(\sum_{n=0}^{\infty} I_n \right) dy \end{aligned}$$

$$\begin{aligned} L(t) = & L(0) + \tau \int_0^t \left(\sum_{n=0}^{\infty} E_n \right) dy - \frac{\tau}{K_L} \int_0^t \left(\sum_{n=0}^{\infty} J_n \right) dy - (v + d_L) \int_0^t \left(\sum_{n=0}^{\infty} L_n \right) dy \\ & - \gamma \int_0^t \left(\sum_{n=0}^{\infty} K_n \right) dy \end{aligned}$$

$$P(t) = P(0) + \nu \int_0^t \left(\sum_{n=0}^{\infty} L_n \right) dy - \frac{\nu}{K_P} \int_0^t \left(\sum_{n=0}^{\infty} M_n \right) dy - (\varepsilon + d_P) \int_0^t \left(\sum_{n=0}^{\infty} P_n \right) dy \\ - \gamma \int_0^t \left(\sum_{n=0}^{\infty} N_n \right) dy$$

$$S_m(t) = S_m(0) + \varepsilon \int_0^t \left(\sum_{n=0}^{\infty} P_n \right) dy - \kappa_1 \int_0^t \left(\sum_{n=0}^{\infty} A_{1n} \right) dy - \kappa_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{2n} \right) dy \\ - \eta \int_0^t \left(\sum_{n=0}^{\infty} S_{mn} \right) dy - \gamma \int_0^t \left(\sum_{n=0}^{\infty} A_{3n} \right) dy$$

$$E_m(t) = E_m(0) + \kappa_1 \int_0^t \left(\sum_{n=0}^{\infty} A_{1n} \right) dy + \kappa_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{2n} \right) dy - (\chi + \eta) \int_0^t \left(\sum_{n=0}^{\infty} E_{mn} \right) dy \\ - \gamma \int_0^t \left(\sum_{n=0}^{\infty} A_{4n} \right) dy$$

$$I_m(t) = I_m(0) + \chi \int_0^t \left(\sum_{n=0}^{\infty} E_{mn} \right) dy - \eta \int_0^t \left(\sum_{n=0}^{\infty} I_{mn} \right) dy - \gamma \int_0^t \left(\sum_{n=0}^{\infty} A_{5n} \right) dy$$

For Awareness Campaign Dynamics:

$$A(t) = A(0) + \omega \int_0^t \left(\sum_{n=0}^{\infty} I_{mn} \right) dy + \delta \int_0^t \left(\sum_{n=0}^{\infty} I_{hn} \right) dy - \theta \int_0^t \left(\sum_{n=0}^{\infty} A_n \right) dy$$

where:

$$S_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n S_{hu(k)} I_{m(k)} \lambda^k \right]_{\lambda=0}, \quad B_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k S_{hu(k)} \lambda^k \right]_{\lambda=0},$$

$$C_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n \frac{S_{ha(k)} I_{m(k)}}{1 + A_k} \lambda^k \right]_{\lambda=0}, \quad D_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k I_{h(k)} \lambda^k \right]_{\lambda=0},$$

$$F_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n E_k S_{m(k)} \lambda^k \right]_{\lambda=0}, \quad G_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n E_k E_{m(k)} \lambda^k \right]_{\lambda=0},$$

$$H_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n E_k I_{m(k)} \lambda^k \right]_{\lambda=0}, \quad I_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k E_k \lambda^k \right]_{\lambda=0},$$

$$J_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n L_k E_k \lambda^k \right]_{\lambda=0}, \quad K_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k L_k \lambda^k \right]_{\lambda=0},$$

$$M_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n P_k L_k \lambda^k \right]_{\lambda=0}, \quad N_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k P_k \lambda^k \right]_{\lambda=0},$$

$$A_{1n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n S_{mk} I_{hk} \lambda^k \right]_{\lambda=0}, \quad A_{2n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n S_{mk} R_{hk} \lambda^k \right]_{\lambda=0},$$

$$A_{3n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n S_{mk} A_k \lambda^k \right]_{\lambda=0}, \quad A_{4n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k E_{mk} \lambda^k \right]_{\lambda=0},$$

$$A_{5n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n A_k I_{mk} \lambda^k \right]_{\lambda=0}, \quad A_{6n} = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n \frac{S_{ha(k)}}{1 + A_k} \lambda^k \right]_{\lambda=0}.$$

A simple observation for the initial stage produces the following

$$\begin{aligned} S_{hu(0)}(t) &= S_{hu}(0) + b_h t; & S_{ha(0)}(t) &= S_{ha}(0); & E_{h(0)}(t) &= E_h(0); \\ I_{h(0)}(t) &= I_h(0); & R_{h(0)}(t) &= R_h(0); & E_0(t) &= E(0); \\ L_0(t) &= L(0); & P_0(t) &= P(0); & S_{m0}(t) &= S_m(0); \\ E_{m0}(t) &= E_m(0); & I_{m0}(t) &= I_m(0); & A_0(t) &= A(0); \end{aligned}$$

Then the recursive formulas are written as

For Human Population Dynamics:

$$\begin{aligned} S_{hu(n+1)}(t) = & -\beta_2 \int_0^t \left(\sum_{n=0}^{\infty} S_n \right) dy + (1 - \xi) \varphi \int_0^t \left(\sum_{n=0}^{\infty} R_{hn} \right) dy \\ & - \alpha_1 \int_0^t \left(\sum_{n=0}^{\infty} B_n \right) dy + \alpha_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{6n} \right) dy + (1 - q) \lambda \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy \\ & - \mu \int_0^t \left(\sum_{n=0}^{\infty} S_{hu(n)} \right) dy \end{aligned}$$

$$\begin{aligned} S_{ha(n+1)}(t) = & -\beta_1 \int_0^t \left(\sum_{n=0}^{\infty} C_n \right) dy + \xi \varphi \int_0^t \left(\sum_{n=0}^{\infty} R_{hn} \right) dy + \alpha_1 \int_0^t \left(\sum_{n=0}^{\infty} B_n \right) dy \\ & - \alpha_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{6n} \right) dy + q \lambda \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy - \mu \int_0^t \left(\sum_{n=0}^{\infty} S_{ha(n)} \right) dy \end{aligned}$$

$$E_{h(n+1)}(t) = \beta_1 \int_0^t \left(\sum_{n=0}^{\infty} C_n \right) dy + \beta_2 \int_0^t \left(\sum_{n=0}^{\infty} S_n \right) dy - (\mu + \lambda + \rho) \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy$$

$$I_{h(n+1)}(t) = \rho \int_0^t \left(\sum_{n=0}^{\infty} E_{hn} \right) dy - (\mu + \sigma) \int_0^t \left(\sum_{n=0}^{\infty} I_{hn} \right) dy - r \int_0^t \left(\sum_{n=0}^{\infty} D_n \right) dy$$

$$R_{h(n+1)}(t) = r \int_0^t \left(\sum_{n=0}^{\infty} D_n \right) dy - (\mu + \varphi) \int_0^t \left(\sum_{n=0}^{\infty} R_{hn} \right) dy$$

For Mosquito Population Dynamics:

$$\begin{aligned} E_{(n+1)}(t) = & b_m \int_0^t \left(\sum_{n=0}^{\infty} (S_{mn} + E_{mn} + I_{mn}) \right) dy - \frac{b_m}{K_E} \int_0^t \left(\sum_{n=0}^{\infty} F_n \right) dy \\ & - \frac{b_m}{K_E} \int_0^t \left(\sum_{n=0}^{\infty} G_n \right) dy - \frac{b_m}{K_E} \int_0^t \left(\sum_{n=0}^{\infty} H_n \right) dy - (\tau + d_E) \int_0^t \left(\sum_{n=0}^{\infty} E_n \right) dy \\ & - \gamma \int_0^t \left(\sum_{n=0}^{\infty} I_n \right) dy \end{aligned}$$

$$L_{(n+1)}(t) = \tau \int_0^t \left(\sum_{n=0}^{\infty} E_n \right) dy - \frac{\tau}{K_L} \int_0^t \left(\sum_{n=0}^{\infty} J_n \right) dy - (v + d_L) \int_0^t \left(\sum_{n=0}^{\infty} L_n \right) dy \\ - \gamma \int_0^t \left(\sum_{n=0}^{\infty} K_n \right) dy$$

$$P_{(n+1)}(t) = v \int_0^t \left(\sum_{n=0}^{\infty} L_n \right) dy - \frac{v}{K_P} \int_0^t \left(\sum_{n=0}^{\infty} M_n \right) dy - (\varepsilon + d_P) \int_0^t \left(\sum_{n=0}^{\infty} P_n \right) dy \\ - \gamma \int_0^t \left(\sum_{n=0}^{\infty} N_n \right) dy$$

$$S_{m(n+1)}(t) = \varepsilon \int_0^t \left(\sum_{n=0}^{\infty} P_n \right) dy - \kappa_1 \int_0^t \left(\sum_{n=0}^{\infty} A_{1n} \right) dy - \kappa_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{2n} \right) dy \\ - \eta \int_0^t \left(\sum_{n=0}^{\infty} S_{mn} \right) dy - \gamma \int_0^t \left(\sum_{n=0}^{\infty} A_{3n} \right) dy$$

$$E_{m(n+1)}(t) = \kappa_1 \int_0^t \left(\sum_{n=0}^{\infty} A_{1n} \right) dy + \kappa_2 \int_0^t \left(\sum_{n=0}^{\infty} A_{2n} \right) dy - (\chi + \eta) \int_0^t \left(\sum_{n=0}^{\infty} E_{mn} \right) dy \\ - \gamma \int_0^t \left(\sum_{n=0}^{\infty} A_{4n} \right) dy$$

$$I_{m(n+1)}(t) = \chi \int_0^t \left(\sum_{n=0}^{\infty} E_{mn} \right) dy - \eta \int_0^t \left(\sum_{n=0}^{\infty} I_{mn} \right) dy - \gamma \int_0^t \left(\sum_{n=0}^{\infty} A_{5n} \right) dy$$

For Awareness Campaign Dynamics:

$$A_{(n+1)}(t) = \omega \int_0^t \left(\sum_{n=0}^{\infty} I_{mn} \right) dy + \delta \int_0^t \left(\sum_{n=0}^{\infty} I_{hn} \right) dy - \theta \int_0^t \left(\sum_{n=0}^{\infty} A_n \right) dy,$$

wherer $n = 0, 1, 2, 3, \dots$

By using the initial conditions $S_{hu}(0) = 15000$; $S_{ha}(0) = 0$; $E_h(0) = 1250$; $I_h(0) = 221$; $R_h(0) = 0$; $E(0) = 2400$; $L(0) = 800$; $P(0) = 400$; $S_m(0) = 300$; $E_m(0) = 30$; $I_m(0) = 50$; $A(0) = 0$; and according to the model from [33], with the values of the parameters in Table 3, the Adomian decomposition solution series are obtained through Python version 3.10.5 software. The solution of each compartment after 8 iterations is given as:

Unaware Susceptible Humans

$$S_{hu} \simeq 15000 - 405589.8t + 5514480.5t^2 - 49697922t^3 + 3.323603 \times 10^8 t^4 \\ - 1.74964 \times 10^9 t^5 + 7.503173 \times 10^9 t^6 - 2.673169 \times 10^{10} t^7 + 7.976186 \times 10^{10} t^8$$

Aware Susceptible Humans

$$S_{ha} \simeq 0.276165t + 107.37401t^2 - 1547.3204t^3 + 14524.859t^4 - 101692.57t^5 \\ + 573476.55t^6 - 2763998.2t^7 + 12119035t^8$$

Exposed Humans

$$E_h \simeq 1250 + 406227.67t - 5527780.6t^2 + 49819189t^3 - 3.33184 \times 10^8t^4 \\ + 1.754071 \times 10^9t^5 - 7.522741 \times 10^9t^6 + 2.680428 \times 10^{10}t^7 - 7.999168 \times 10^{10}t^8$$

Infected Humans

$$I_h \simeq 221 + 57.422579t + 13087.233t^2 - 120191.25t^3 + 809255.04t^4 - 4323584.2t^5 \\ + 18938188t^6 - 69466819t^7 + 2.158434 \times 10^8t^8$$

Recovered Humans

$$R_h \simeq 102.32924t^2 + 2.781025t^3 + 3120.5752t^4 - 23214.195t^5 + 133396.76t^6 \\ - 649279.11t^7 + 2793322.8t^8$$

Mosquito Eggs

$$E \simeq 2400 + 2898.0268t - 5056.6017t^2 - 4447.8181t^3 + 1774.9697t^4 + 26494.184t^5 \\ - 89663.95t^6 + 277229t^7 - 792365.06t^8$$

Mosquito Larvae

$$L \simeq 800 + 403.77344t - 1023.8076t^2 - 897.72912t^3 - 576.8477t^4 + 9683.408t^5 \\ - 36213.193t^6 + 127417.66t^7 - 400757.99t^8$$

Mosquito Pupae

$$P \simeq 400 + 420.46909t - 674.6931t^2 - 648.95282t^3 - 96.707269t^4 + 4503.451t^5 \\ - 15332.431t^6 + 50710.785t^7 - 150928.93t^8$$

Susceptible Mosquitoes

$$S_m \simeq 300 - 17.2366t - 507.41292t^2 - 2350.1991t^3 + 15994.135t^4 - 80992.876t^5 \\ + 365524.7t^6 - 1436092.1t^7 + 5165162t^8$$

Exposed Mosquitoes

$$E_m \simeq 30 + 113.38104t - 47.350427t^2 + 2191.5149t^3 - 16097.534t^4 + 85012.551t^5 \\ - 381929.9t^6 + 1493814.8t^7 - 5346186.2t^8$$

Infected Mosquitoes

$$I_m \simeq 50 + 1.29098t - 88.058678t^2 - 0.282463t^3 + 32.625632t^4 + 381.31221t^5 \\ - 1663.3636t^6 + 5622.5948t^7 - 16835.394t^8$$

Awareness Programs

$$A \simeq 8.297551t - 0.263579t^2 + 14.819219t^3 - 132.11092t^4 + 713.26684t^5 \\ - 3162.1911t^6 + 11873.102t^7 - 38120.336t^8$$

5. Application of the Homotopy Perturbation Method

Reverting to the malaria model under consideration, we obtain the following after some rearrangement

Human Population Dynamics:

$$\frac{dS_{hu}}{dt} - b_h + \beta_2 S_{hu} I_m - (1 - \xi)\varphi R_h + \alpha_1 A S_{hu} - \frac{\alpha_2 S_{ha}}{1 + A} - (1 - q)\lambda E_h + \mu S_{hu} = 0 \quad (5.1)$$

$$\frac{dS_{ha}}{dt} + \frac{\beta_1 S_{ha} I_m}{1 + A} - \xi\varphi R_h - \alpha_1 A S_{hu} + \frac{\alpha_2 S_{ha}}{1 + A} - q\lambda E_h + \mu S_{ha} = 0 \quad (5.2)$$

$$\frac{dE_h}{dt} - \frac{\beta_1 S_{ha} I_m}{1 + A} - \beta_2 S_{hu} I_m + (\mu + \lambda + \rho)E_h = 0 \quad (5.3)$$

$$\frac{dI_h}{dt} - \rho E_h + (\mu + \sigma + rA)I_h = 0 \quad (5.4)$$

$$\frac{dR_h}{dt} - rA I_h + (\mu + \varphi)R_h = 0 \quad (5.5)$$

Mosquito Population Dynamics:

$$\frac{dE}{dt} - b_m \left(1 - \frac{E}{K_E}\right) N_m + (\tau + d_E + \gamma A)E = 0 \quad (5.6)$$

$$\frac{dL}{dt} - \tau \left(1 - \frac{L}{K_L}\right) E + (v + d_L + \gamma A)L = 0 \quad (5.7)$$

$$\frac{dP}{dt} - v \left(1 - \frac{P}{K_P}\right) L + (\varepsilon + d_P + \gamma A)P = 0 \quad (5.8)$$

$$\frac{dS_m}{dt} - \varepsilon P + \kappa_1 S_m I_h + \kappa_2 S_m R_h + (\eta + \gamma A)S_m = 0 \quad (5.9)$$

$$\frac{dE_m}{dt} - \kappa_1 S_m I_h - \kappa_2 S_m R_h + (\chi + \eta + \gamma A)E_m = 0 \quad (5.10)$$

$$\frac{dI_m}{dt} - \chi E_m + (\eta + \gamma A)I_m = 0 \quad (5.11)$$

Awareness Campaign Dynamics:

$$\frac{dA}{dt} - \omega I_m - \delta I_h + \theta A = 0 \quad (5.12)$$

Now, we define the Homotopy for each of the equations above:

$$H(S_{hu}, p) = (1 - p) \left[\frac{dS_{hu}}{dt} - \frac{dS_{hu0}}{dt} \right] + p \left[\frac{dS_{hu}}{dt} - b_h + \beta_2 S_{hu} I_m - (1 - \xi) \varphi R_h \right. \quad (5.13)$$

$$\left. + \alpha_1 A S_{hu} - \frac{\alpha_2 S_{ha}}{1 + A} - (1 - q) \lambda E_h + \mu S_{hu} \right] = 0 \quad (5.14)$$

$$H(S_{ha}, p) = (1 - p) \left[\frac{dS_{ha}}{dt} - \frac{dS_{ha0}}{dt} \right] + p \left[\frac{dS_{ha}}{dt} + \frac{\beta_1 S_{ha} I_m}{1 + A} - \xi \varphi R_h \right. \quad (5.15)$$

$$\left. - \alpha_1 A S_{hu} + \frac{\alpha_2 S_{ha}}{1 + A} - q \lambda E_h + \mu S_{ha} \right] = 0 \quad (5.16)$$

$$H(E_h, p) = (1 - p) \left[\frac{dE_h}{dt} - \frac{dE_{h0}}{dt} \right] + p \left[\frac{dE_h}{dt} - \frac{\beta_1 S_{ha} I_m}{1 + A} - \beta_2 S_{hu} I_m + (\mu + \lambda + \rho) E_h \right] = 0 \quad (5.17)$$

$$H(I_h, p) = (1 - p) \left[\frac{dI_h}{dt} - \frac{dI_{h0}}{dt} \right] + p \left[\frac{dI_h}{dt} - \rho E_h + (\mu + \sigma + rA) I_h \right] = 0 \quad (5.18)$$

$$H(R_h, p) = (1 - p) \left[\frac{dR_h}{dt} - \frac{dR_{h0}}{dt} \right] + p \left[\frac{dR_h}{dt} - rA I_h + (\mu + \varphi) R_h \right] = 0, \quad (5.19)$$

$$H(E, p) = (1-p) \left[\frac{dE}{dt} - \frac{dE_0}{dt} \right] + p \left[\frac{dE}{dt} - b_m \left(1 - \frac{E}{K_E} \right) N_m + (\tau + d_E + \gamma A) E \right] = 0 \quad (5.20)$$

$$H(L, p) = (1-p) \left[\frac{dL}{dt} - \frac{dL_0}{dt} \right] + p \left[\frac{dL}{dt} - \tau \left(1 - \frac{L}{K_L} \right) E + (v + d_L + \gamma A) L \right] = 0 \quad (5.21)$$

$$H(P, p) = (1-p) \left[\frac{dP}{dt} - \frac{dP_0}{dt} \right] + p \left[\frac{dP}{dt} - v \left(1 - \frac{P}{K_P} \right) L + (\varepsilon + d_P + \gamma A) P \right] = 0 \quad (5.22)$$

$$H(S_m, p) = (1-p) \left[\frac{dS_m}{dt} - \frac{dS_{m0}}{dt} \right] + p \left[\frac{dS_m}{dt} - \varepsilon P + \kappa_1 S_m I_h + \kappa_2 S_m R_h + (\eta + \gamma A) S_m \right] = 0 \quad (5.23)$$

$$H(E_m, p) = (1-p) \left[\frac{dE_m}{dt} - \frac{dE_{m0}}{dt} \right] + p \left[\frac{dE_m}{dt} - \kappa_1 S_m I_h - \kappa_2 S_m R_h + (\chi + \eta + \gamma A) E_m \right] = 0 \quad (5.24)$$

$$H(I_m, p) = (1-p) \left[\frac{dI_m}{dt} - \frac{dI_{m0}}{dt} \right] + p \left[\frac{dI_m}{dt} - \chi E_m + (\eta + \gamma A) I_m \right] = 0 \quad (5.25)$$

$$H(A, p) = (1-p) \left[\frac{dA}{dt} - \frac{dA_0}{dt} \right] + p \left[\frac{dA}{dt} - \omega I_m - \delta I_h + \theta A \right] = 0 \quad (5.26)$$

We can further evaluate the homotopy above to give:

$$H(S_{hu}, p) = \frac{dS_{hu}}{dt} - \frac{dS_{hu0}}{dt} + p \frac{dS_{hu0}}{dt} + p \left[-b_h + \beta_2 S_{hu} I_m - (1 - \xi) \varphi R_h + \alpha_1 A S_{hu} - \frac{\alpha_2 S_{ha}}{1 + A} - (1 - q) \lambda E_h + \mu S_{hu} \right] = 0 \quad (5.27)$$

$$H(S_{ha}, p) = \frac{dS_{ha}}{dt} - \frac{dS_{ha0}}{dt} + p \frac{dS_{ha0}}{dt} + p \left[\frac{\beta_1 S_{ha} I_m}{1 + A} - \xi \varphi R_h - \alpha_1 A S_{hu} + \frac{\alpha_2 S_{ha}}{1 + A} - q \lambda E_h + \mu S_{ha} \right] = 0 \quad (5.28)$$

$$H(E_h, p) = \frac{dE_h}{dt} - \frac{dE_{h0}}{dt} + p \frac{dE_{h0}}{dt} + p \left[-\frac{\beta_1 S_{ha} I_m}{1 + A} - \beta_2 S_{hu} I_m + (\mu + \lambda + \rho) E_h \right] = 0 \quad (5.29)$$

$$H(I_h, p) = \frac{dI_h}{dt} - \frac{dI_{h0}}{dt} + p \frac{dI_{h0}}{dt} + p \left[-\rho E_h + (\mu + \sigma + r A) I_h \right] = 0 \quad (5.30)$$

$$H(R_h, p) = \frac{dR_h}{dt} - \frac{dR_{h0}}{dt} + p \frac{dR_{h0}}{dt} + p \left[-r A I_h + (\mu + \varphi) R_h \right] = 0 \quad (5.31)$$

$$H(E, p) = \frac{dE}{dt} - \frac{dE_0}{dt} + p \frac{dE_0}{dt} + p \left[-b_m \left(1 - \frac{E}{K_E} \right) N_m + (\tau + d_E + \gamma A) E \right] = 0 \quad (5.32)$$

$$H(L, p) = \frac{dL}{dt} - \frac{dL_0}{dt} + p \frac{dL_0}{dt} + p \left[-\tau \left(1 - \frac{L}{K_L} \right) E + (v + d_L + \gamma A) L \right] = 0 \quad (5.33)$$

$$H(P, p) = \frac{dP}{dt} - \frac{dP_0}{dt} + p \frac{dP_0}{dt} + p \left[-v \left(1 - \frac{P}{K_P} \right) L + (\varepsilon + d_P + \gamma A) P \right] = 0 \quad (5.34)$$

$$H(S_m, p) = \frac{dS_m}{dt} - \frac{dS_{m0}}{dt} + p \frac{dS_{m0}}{dt} + p [-\varepsilon P + \kappa_1 S_m I_h + \kappa_2 S_m R_h + (\eta + \gamma A) S_m] = 0 \quad (5.35)$$

$$H(E_m, p) = \frac{dE_m}{dt} - \frac{dE_{m0}}{dt} + p \frac{dE_{m0}}{dt} + p [-\kappa_1 S_m I_h - \kappa_2 S_m R_h + (\chi + \eta + \gamma A) E_m] = 0 \quad (5.36)$$

$$H(I_m, p) = \frac{dI_m}{dt} - \frac{dI_{m0}}{dt} + p \frac{dI_{m0}}{dt} + p [-\chi E_m + (\eta + \gamma A) I_m] = 0 \quad (5.37)$$

$$H(A, p) = \frac{dA}{dt} - \frac{dA_0}{dt} + p \frac{dA_0}{dt} + p [-\omega I_m - \delta I_h + \theta A] = 0 \quad (5.38)$$

Now, we assume the power series solution as:

$$S_{hu}(t) = S_{hu0}(t) + p S_{hu1}(t) + p^2 S_{hu2}(t) + p^3 S_{hu3}(t) + \dots \quad (5.39)$$

$$S_{ha}(t) = S_{ha0}(t) + p S_{ha1}(t) + p^2 S_{ha2}(t) + p^3 S_{ha3}(t) + \dots \quad (5.40)$$

$$E_h(t) = E_{h0}(t) + p E_{h1}(t) + p^2 E_{h2}(t) + p^3 E_{h3}(t) + \dots \quad (5.41)$$

$$I_h(t) = I_{h0}(t) + p I_{h1}(t) + p^2 I_{h2}(t) + p^3 I_{h3}(t) + \dots \quad (5.42)$$

$$R_h(t) = R_{h0}(t) + p R_{h1}(t) + p^2 R_{h2}(t) + p^3 R_{h3}(t) + \dots \quad (5.43)$$

$$E(t) = E_0(t) + p E_1(t) + p^2 E_2(t) + p^3 E_3(t) + \dots \quad (5.44)$$

$$L(t) = L_0(t) + p L_1(t) + p^2 L_2(t) + p^3 L_3(t) + \dots \quad (5.45)$$

$$P(t) = P_0(t) + p P_1(t) + p^2 P_2(t) + p^3 P_3(t) + \dots \quad (5.46)$$

$$S_m(t) = S_{m0}(t) + p S_{m1}(t) + p^2 S_{m2}(t) + p^3 S_{m3}(t) + \dots \quad (5.47)$$

$$E_m(t) = E_{m0}(t) + p E_{m1}(t) + p^2 E_{m2}(t) + p^3 E_{m3}(t) + \dots \quad (5.48)$$

$$I_m(t) = I_{m0}(t) + p I_{m1}(t) + p^2 I_{m2}(t) + p^3 I_{m3}(t) + \dots \quad (5.49)$$

$$A(t) = A_0(t) + p A_1(t) + p^2 A_2(t) + p^3 A_3(t) + \dots \quad (5.50)$$

Now, if we substitute (5.39) - (5.50) into (5.27) - (5.38) for the respective compartment, we obtain a complicated expansion for each of the compartments. However, the results obtained for each compartment using the HPM were further evaluated using the Python version 3.10.5 software for 8 iterations. The solution of each compartment after 8 iterations is given as:

Unaware Susceptible Humans (S_{hu}):

$$\begin{aligned} S_{hu} \simeq & 15000.0 - 406219.8t + 5521796.7t^2 - 49753119.0t^3 + 3.3265557 \times 10^8 t^4 - 1.7508057 \times 10^9 t^5 \\ & + 7.506505 \times 10^9 t^6 - 2.673762 \times 10^{10} t^7 + 7.9761862 \times 10^{10} t^8 + 1.365286 \times 10^{11} t^9 \\ & - 4.6187551 \times 10^{11} t^{10} - 1.8754129 \times 10^{11} t^{11} + 5.3832069 \times 10^{11} t^{12} + 2.1835669 \times 10^{12} t^{13} \\ & - 3.6382001 \times 10^{12} t^{14} - 6.4723095 \times 10^{12} t^{15} + 3.3767615 \times 10^{13} t^{16} - 3.966857 \times 10^{13} t^{17} \\ & - 1.210606 \times 10^{13} t^{18} + 1.3480529 \times 10^{14} t^{19} - 1.6577525 \times 10^{15} t^{20} \end{aligned}$$

Aware Susceptible Humans (S_{ha}):

$$\begin{aligned} S_{ha} \simeq & 0.27616451t + 107.37401t^2 - 1549.0203t^3 + 14537.602t^4 - 101760.0t^5 + 573733.6t^6 \\ & - 2764638.4t^7 + 12119035.0t^8 - 49417811.0t^9 + 1.3946591 \times 10^8 t^{10} - 9.9752831 \times 10^8 t^{11} \\ & + 7.2275509 \times 10^9 t^{12} - 5.1500414 \times 10^{10} t^{13} + 3.987727 \times 10^{11} t^{14} - 3.2510243 \times 10^{12} t^{15} \\ & + 2.7060319 \times 10^{13} t^{16} - 2.2732915 \times 10^{14} t^{17} + 1.9136505 \times 10^{15} t^{18} - 1.6149571 \times 10^{16} t^{19} \\ & + 1.3666212 \times 10^{17} t^{20} \end{aligned}$$

Exposed Humans (E_h):

$$\begin{aligned} E_h \simeq & 1250.0 + 406227.67t - 5535096.7t^2 + 49874519.0t^3 - 3.3347997 \times 10^8 t^4 + 1.7552397 \times 10^9 t^5 \\ & - 7.5260813 \times 10^9 t^6 + 2.6810225 \times 10^{10} t^7 - 7.9991684 \times 10^{10} t^8 - 1.367024 \times 10^{11} t^9 \\ & + 4.6251683 \times 10^{11} t^{10} + 1.8879228 \times 10^{11} t^{11} - 5.4639544 \times 10^{11} t^{12} - 2.1338344 \times 10^{12} t^{13} \\ & + 3.2412872 \times 10^{12} t^{14} + 9.7329909 \times 10^{12} t^{15} - 6.0852333 \times 10^{13} t^{16} + 2.6709053 \times 10^{14} t^{17} \\ & - 1.9023098 \times 10^{15} t^{18} + 1.6020904 \times 10^{16} t^{19} - 1.3505344 \times 10^{17} t^{20} \end{aligned}$$

Infected Humans (I_h):

$$\begin{aligned} I_h \simeq & 221.0 + 57.422579t + 13087.233t^2 - 120323.25t^3 + 809976.65t^4 - 4326464.4t^5 + 18946599.0t^6 \\ & - 69482239.0t^7 + 2.1584336 \times 10^8 t^8 + 2.3482764 \times 10^8 t^9 - 8.0046477 \times 10^8 t^{10} \\ & - 1.6260102 \times 10^8 t^{11} + 4.3015302 \times 10^8 t^{12} + 3.0350308 \times 10^9 t^{13} - 4.6240288 \times 10^9 t^{14} \\ & - 6.9777647 \times 10^9 t^{15} + 3.7204482 \times 10^{10} t^{16} - 1.1344948 \times 10^{11} t^{17} + 7.4911045 \times 10^{11} t^{18} \\ & - 6.1146566 \times 10^{12} t^{19} + 4.910701 \times 10^{13} t^{20} \end{aligned}$$

Recovered Humans (R_h):

$$\begin{aligned} R_h \simeq & 102.32924t^2 + 2.7810248t^3 + 3120.5752t^4 - 23229.292t^5 + 133454.58t^6 - 649423.03t^7 \\ & + 2793322.8t^8 - 11006073.0t^9 + 17594252.0t^{10} - 91192265.0t^{11} + 4.1816295 \times 10^8 t^{12} \\ & - 1.2635486 \times 10^9 t^{13} + 2.7614076 \times 10^9 t^{14} - 2.6895709 \times 10^9 t^{15} - 1.2788262 \times 10^{10} t^{16} \\ & + 2.0578525 \times 10^{10} t^{17} + 1.6771098 \times 10^{10} t^{18} - 2.7515933 \times 10^{10} t^{19} - 9.9506248 \times 10^9 t^{20} \end{aligned}$$

Mosquito Eggs (E):

$$\begin{aligned} E \simeq & 2400.0 + 2898.0268t - 5056.6017t^2 - 4447.8181t^3 + 1774.9697t^4 + 26512.558t^5 - 89705.861t^6 \\ & + 277289.4t^7 - 792365.06t^8 - 3025620.0t^9 + 3191651.3t^{10} + 4770797.8t^{11} + 2458704.3t^{12} \\ & - 2.8240611 \times 10^7 t^{13} + 3.5070448 \times 10^7 t^{14} + 2.5725114 \times 10^7 t^{15} - 3.979147 \times 10^8 t^{16} \\ & + 2.1024622 \times 10^8 t^{17} + 6.2319541 \times 10^8 t^{18} + 9.4608368 \times 10^8 t^{19} - 6.9377398 \times 10^9 t^{20} \end{aligned}$$

Mosquito Larvae (L):

$$\begin{aligned} L \simeq & 800.0 + 403.77344t - 1023.8076t^2 - 897.72912t^3 - 576.8477t^4 + 9689.5327t^5 - 36229.888t^6 \\ & + 127445.82t^7 - 400757.99t^8 - 590559.77t^9 + 1020918.2t^{10} + 438605.45t^{11} + 2118462.6t^{12} \\ & - 1.1007381 \times 10^7 t^{13} + 1.9862659 \times 10^7 t^{14} - 1.4425663 \times 10^7 t^{15} - 1.0228739 \times 10^8 t^{16} \\ & + 1.194976 \times 10^8 t^{17} + 3.6230011 \times 10^7 t^{18} + 4.5301229 \times 10^8 t^{19} - 2.5966211 \times 10^9 t^{20} \end{aligned}$$

Mosquito Pupae (P):

$$\begin{aligned} P \simeq & 400.0 + 420.46909t - 674.6931t^2 - 648.95282t^3 - 96.707269t^4 + 4506.5133t^5 - 15339.652t^6 \\ & + 50721.889t^7 - 150928.93t^8 - 439400.77t^9 + 492494.18t^{10} + 579757.48t^{11} + 712985.19t^{12} \\ & - 4756378.5t^{13} + 6675979.3t^{14} + 544664.31t^{15} - 6.0141789 \times 10^7 t^{16} + 4.0892834 \times 10^7 t^{17} \\ & + 7.0850655 \times 10^7 t^{18} + 1.8386063 \times 10^8 t^{19} - 1.1701102 \times 10^9 t^{20} \end{aligned}$$

Susceptible Mosquitoes (S_m):

$$\begin{aligned} S_m \simeq & 300.0 - 17.2366t - 507.41292t^2 - 2350.1991t^3 + 16007.94t^4 - 81048.475t^5 + 365682.71t^6 \\ & - 1436410.6t^7 + 5165162.0t^8 - 3165955.8t^9 + 2.6632310 \times 10^7 t^{10} - 2.1152236 \times 10^8 t^{11} \\ & + 8.1362641 \times 10^8 t^{12} - 2.5496042 \times 10^9 t^{13} + 7.2010691 \times 10^9 t^{14} - 1.7434591 \times 10^{10} t^{15} \\ & + 3.7870984 \times 10^{10} t^{16} - 2.0444951 \times 10^{11} t^{17} + 8.6334584 \times 10^{11} t^{18} - 2.247335 \times 10^{12} t^{19} \\ & + 4.6011352 \times 10^{12} t^{20} \end{aligned}$$

Exposed Mosquitoes (E_m):

$$\begin{aligned} E_m \simeq & 30.0 + 113.38104t - 47.350427t^2 + 2191.5149t^3 - 16111.338t^4 + 85070.803t^5 - 382095.55t^6 \\ & + 1494146.1t^7 - 5346186.2t^8 + 2929643.3t^9 - 2.6115814 \times 10^7 t^{10} + 2.1181924 \times 10^8 t^{11} \\ & - 8.1367834 \times 10^8 t^{12} + 2.5469592 \times 10^9 t^{13} - 7.1957836 \times 10^9 t^{14} + 1.7429915 \times 10^{10} t^{15} \\ & - 3.7936421 \times 10^{10} t^{16} + 2.0461631 \times 10^{11} t^{17} - 8.6355038 \times 10^{11} t^{18} + 2.2477083 \times 10^{12} t^{19} \\ & - 4.6021949 \times 10^{12} t^{20} \end{aligned}$$

Infected Mosquitoes (I_m):

$$\begin{aligned} I_m \simeq & 50.0 + 1.29098t - 88.058678t^2 - 0.28246297t^3 + 32.625632t^4 + 381.56755t^5 - 1664.1276t^6 \\ & + 5623.8621t^7 - 16835.394t^8 - 29273.027t^9 + 107007.92t^{10} - 212860.54t^{11} + 798061.27t^{12} \\ & - 2512916.3t^{13} + 4951745.8t^{14} - 2743461.7t^{15} + 1.4361512 \times 10^7 t^{16} - 9.4949173 \times 10^7 t^{17} \\ & + 2.17627178 \times 10^8 t^{18} - 1.923841 \times 10^8 t^{19} - 1.9133124 \times 10^8 t^{20} \end{aligned}$$

Awareness Programs (A):

$$\begin{aligned} A \simeq & 8.297551t - 0.2635791t^2 + 14.819219t^3 - 132.22658t^4 + 713.74307t^5 - 3163.5952t^6 \\ & + 11875.738t^7 - 38120.336t^8 - 24288.84t^9 + 84811.932t^{10} + 13887.148t^{11} - 47259.162t^{12} \\ & - 141731.93t^{13} + 116213.54t^{14} + 420607.47t^{15} - 759328.5t^{16} + 2520192.4t^{17} \\ & - 2.0801080 \times 10^7 t^{18} + 1.5812763 \times 10^8 t^{19} - 1.1855129 \times 10^9 t^{20} \end{aligned}$$

6. Numerical Results and Discussions

Numerical results were obtained by applying the analytical results obtained from both the ADM and the HPM from sections 4 and 5 of this paper, respectively. The values for the parameters are described below:

Table 3: Table of Values of Model Parameters [33]

Parameter	Unit	Value	Parameter	Unit	Value
b_h	Humans/day	90	d_L	day ⁻¹	0.071429
α_1	day ⁻¹	0.001005	d_P	day ⁻¹	0.033333
α_2	day ⁻¹	0.003642	τ	day ⁻¹	0.465955
β_1	day ⁻¹	0.006162	ν	day ⁻¹	0.709890
β_2	day ⁻¹	0.541938	κ_1	day ⁻¹	0.001743
μ	day ⁻¹	0.000040891	κ_2	day ⁻¹	0.010966
σ	day ⁻¹	0.107504	b_m	Dimensionless	12.797272
λ	day ⁻¹	0.115671	ε	day ⁻¹	0.254144
r	day ⁻¹	0.111606	γ	day ⁻¹	0.441291
ρ	day ⁻¹	0.064952	K_E	Space	15000
φ	day ⁻¹	0.374557	K_L	Space	10000
q	Dimensionless	0.001910	K_P	Space	7000
ξ	Dimensionless	0.001000	θ	day ⁻¹	0.116657
χ	day ⁻¹	0.061551	δ	day ⁻¹	0.004381
η	day ⁻¹	0.011111	ω	day ⁻¹	0.146587
d_E	day ⁻¹	0.028571			

Table 4: Life Expectancy Parameters [33]

Parameter	Description	Unit	Value
$1/\mu$	Life expectancy of humans	days	24,455
$1/\eta$	Life expectancy of mature mosquitoes	days	90
$1/d_E$	Life expectancy of mosquito eggs	days	35
$1/d_L$	Life expectancy of larvae	days	14
$1/d_P$	Life expectancy of pupae	days	30

The twelve plots revealed below are plots that show the comparison of the three methods, ADM, HPM, and the RK45 method. The RK45 method was run with an inbuilt function known as `solve_ivp` from SciPy. The table of values for the comparison of the results obtained from the three methods is shown before each plot. The errors obtained by the HPM and ADM in comparison with the RK45 method are also displayed in the table to critically observe the convergence of the two methods with the RK45 method.

Additionally, our analysis revealed that the Adomian Decomposition method and the Homotopy Perturbation method can align with the RK45 method for a short period. We decide to plot our graph for a very short amount of time between 0 and 0.1 days with a time step of 0.01. Which is one of the limitations of using semi-analytical methods.

Table 5: Comparison for Unaware Susceptible Humans (S_{hu})

Time	ADM_Value	HPM_Value	RK45_Value	ADM_Error	HPM_Error
0.00	15000.000000	15000.000000	15000.000000	0.000000	0.000000
0.01	11449.008005	11443.387265	11442.808616	6.199389	0.578649
0.02	8744.439647	8734.368323	8734.006020	10.433627	0.362304
0.03	6685.127673	6671.537458	6671.507922	13.619751	0.029536
0.04	5117.465055	5101.118383	5098.078428	19.386627	3.039955
0.05	3924.429093	3906.069228	3904.091951	20.337143	1.977277
0.06	3017.493577	2998.259482	2997.386944	20.106633	0.872538
0.07	2331.207317	2313.674148	2302.539813	28.667505	11.134335
0.08	1820.538981	1810.976222	1773.583128	46.955854	37.393094
0.09	1461.409740	1474.107282	1373.689449	87.720290	100.417832
0.10	1255.156859	1319.912094	1069.268034	185.888824	250.644059

Total Absolute Error (vs RK45): ADM = 439.315642, HPM = 406.449580

CONCLUSION: For S_{hu} , HPM provides a closer approximation.

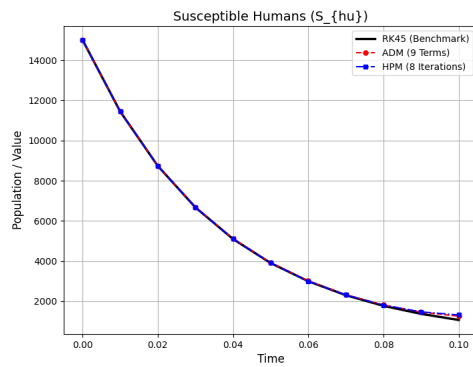


Figure 1: Graph of the Unaware Susceptible Population showing the ADM, HPM and RK45

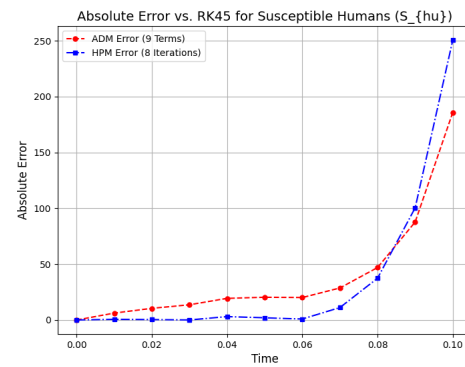


Figure 2: ADM and HPM error plots for the Unaware Susceptible Population vs RK45

Table 6: Comparison for Aware Susceptible Humans (S_{ha})

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	0.000000	0.000000	0.000000	0.000000	0.000000
0.01	0.012087	0.012086	0.012083	0.000005	0.000003
0.02	0.038126	0.038115	0.038118	0.000009	0.000003
0.03	0.072803	0.072766	0.072764	0.000039	0.000001
0.04	0.112562	0.112468	0.112502	0.000060	0.000034
0.05	0.155104	0.154867	0.154897	0.000208	0.000029
0.06	0.199115	0.198432	0.198416	0.000699	0.000016
0.07	0.244253	0.242158	0.242281	0.001972	0.000124
0.08	0.291471	0.285290	0.285829	0.005642	0.000539
0.09	0.343815	0.327040	0.328664	0.015152	0.001624
0.10	0.407863	0.366422	0.370726	0.037138	0.004303
Total Absolute Error (vs RK45): ADM = 0.060922, HPM = 0.006676					
CONCLUSION: For S_{ha} , HPM provides a closer approximation.					

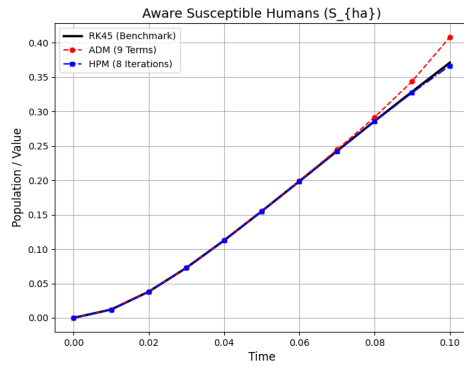


Figure 3: Graph of the Aware Susceptible Population showing the ADM, HPM, and RK45

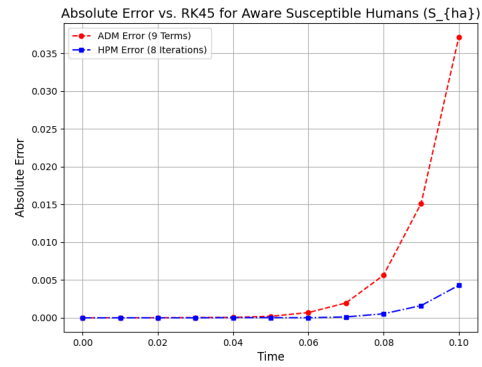


Figure 4: ADM and HPM error plots for the Aware Susceptible Population vs RK45

Table 7: Comparison for Exposed Humans (E_h)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	1250.000000	1250.000000	1250.000000	0.000000	0.000000
0.01	4806.154139	4805.475007	4806.055053	0.099086	0.580046
0.02	7513.849088	7511.321377	7511.684545	2.164543	0.363168
0.03	9574.740255	9569.433560	9569.463174	5.277081	0.029614
0.04	11142.806127	11133.959731	11137.007003	5.799124	3.047272
0.05	12335.351408	12322.223969	12324.205982	11.145426	1.982014
0.06	13241.114603	13222.568740	13223.443347	17.671256	0.874608
0.07	13925.702115	13899.162669	13910.323686	15.378429	11.161017
0.08	14434.248440	14393.441731	14430.924468	3.323972	37.482737
0.09	14790.879493	14721.502432	14822.161161	31.281669	100.658729
0.10	14994.233582	14866.458366	15117.704904	123.471322	251.246538
Total Absolute Error (vs RK45): ADM = 215.611907, HPM = 407.425743					
CONCLUSION: For E_h , ADM provides a closer approximation.					

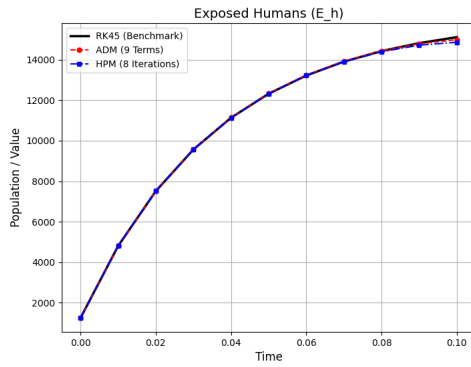


Figure 5: Graph of the Exposed Human Population showing the ADM, HPM, and RK45

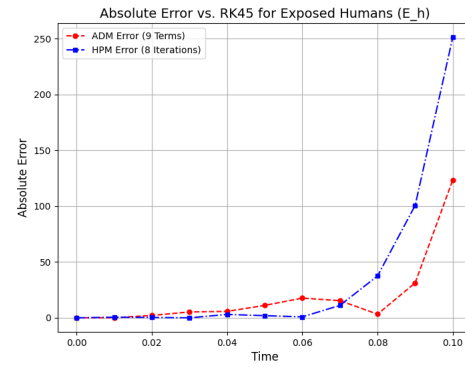


Figure 6: ADM and HPM error plots for the Exposed Human Population vs RK45

Table 8: Comparison for Infected Humans (I_h)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	221.000000	221.000000	221.000000	0.000000	0.000000
0.01	222.770436	222.770311	222.768911	0.001525	0.001400
0.02	226.538589	226.537640	226.536768	0.001821	0.000872
0.03	231.818885	231.815845	231.815769	0.003116	0.000077
0.04	238.240798	238.233987	238.226592	0.014206	0.007395
0.05	245.522099	245.509708	245.504911	0.017187	0.004797
0.06	253.449381	253.430208	253.428149	0.021231	0.002059
0.07	261.865388	261.840651	261.813684	0.051704	0.026967
0.08	270.663576	270.640820	270.550062	0.113514	0.090758
0.09	279.791178	279.791762	279.547630	0.243548	0.244133
0.10	289.262941	289.335006	288.723999	0.538942	0.611007

Total Absolute Error (vs RK45): ADM = 1.006795, HPM = 0.989464

CONCLUSION: For I_h , HPM provides a closer approximation.

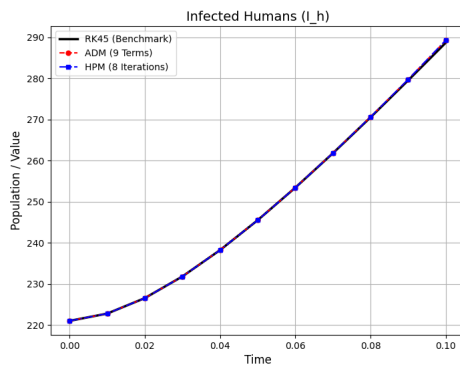


Figure 7: Graph of the Infected Human Population showing the ADM, HPM, and RK45

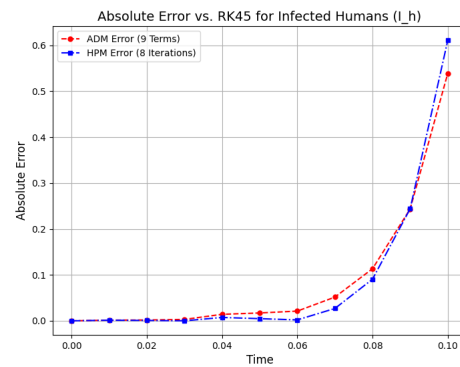


Figure 8: ADM and HPM error plots for the Infected Human Population vs RK45

Table 9: Comparison for Recovered Humans (R_h)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	0.000000	0.000000	0.000000	0.000000	0.000000
0.01	0.010265	0.010265	0.010265	0.000000	0.000000
0.02	0.041387	0.041387	0.041388	0.000001	0.000001
0.03	0.094220	0.094219	0.094219	0.000001	0.000000
0.04	0.169975	0.169971	0.169985	0.000011	0.000015
0.05	0.270106	0.270082	0.270094	0.000013	0.000012
0.06	0.396253	0.396140	0.396138	0.000115	0.000003
0.07	0.550233	0.549806	0.549860	0.000374	0.000054
0.08	0.734121	0.732740	0.732955	0.001166	0.000216
0.09	0.950420	0.946509	0.947150	0.003270	0.000641
0.10	1.202391	1.192441	1.194231	0.008161	0.001790
Total Absolute Error (vs RK45): ADM = 0.013110, HPM = 0.002731					
CONCLUSION: For R_h , HPM provides a closer approximation.					

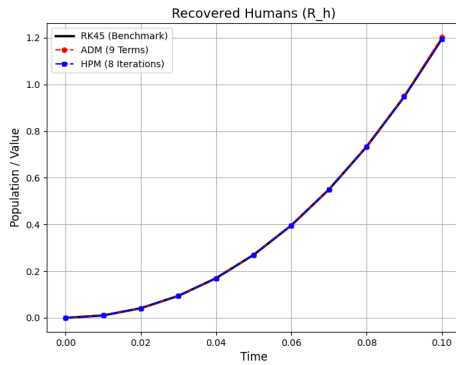


Figure 9: Graph of the Recovered Humans Population showing the ADM, HPM, and RK45

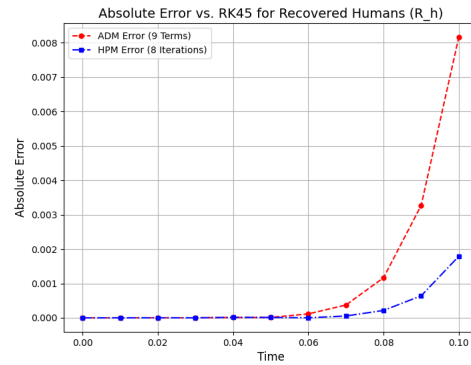


Figure 10: ADM and HPM error plots for the Recovered Human Population vs RK45

Table 10: Comparison for Mosquito Eggs (E)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	2400.000000	2400.000000	2400.000000	0.000000	0.000000
0.01	2428.470181	2428.470181	2428.470190	0.000009	0.000009
0.02	2455.902677	2455.902677	2455.902682	0.000006	0.000005
0.03	2482.271794	2482.271794	2482.271795	0.000001	0.000001
0.04	2507.552780	2507.552781	2507.552832	0.000052	0.000051
0.05	2531.722017	2531.722017	2531.722048	0.000031	0.000031
0.06	2554.757180	2554.757164	2554.757175	0.000004	0.000012
0.07	2576.637351	2576.637265	2576.637445	0.000095	0.000180
0.08	2597.343110	2597.342793	2597.343402	0.000292	0.000609
0.09	2616.856580	2616.855623	2616.857271	0.000691	0.001648
0.10	2635.161421	2635.158910	2635.163085	0.001664	0.004175
Total Absolute Error (vs RK45): ADM = 0.002844, HPM = 0.006721					
CONCLUSION: For E , ADM provides a closer approximation.					

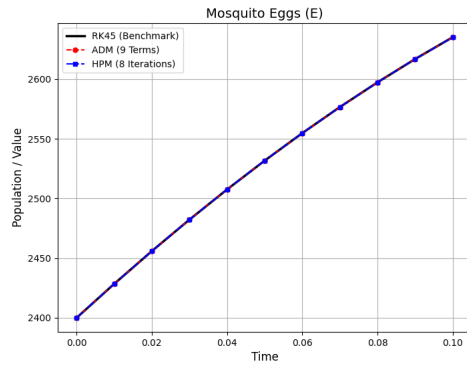


Figure 11: Graph of the Mosquito eggs Population showing the ADM, HPM, and RK45

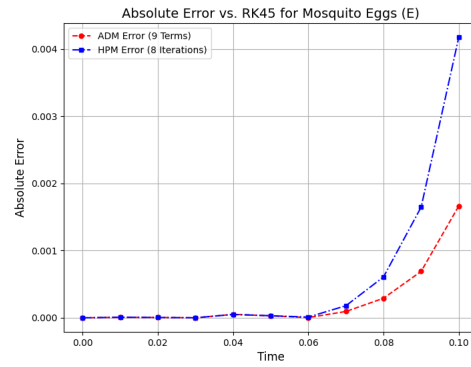


Figure 12: ADM and HPM error plots for the Mosquito eggs Population vs RK45

Table 11: Comparison for Mosquito Larvae (L)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	800.000000	800.000000	800.000000	0.000000	0.000000
0.01	803.934451	803.934451	803.934454	0.000003	0.000003
0.02	807.658700	807.658700	807.658702	0.000002	0.000002
0.03	811.167282	811.167282	811.167282	0.000000	0.000000
0.04	814.454776	814.454776	814.454792	0.000017	0.000016
0.05	817.515876	817.515876	817.515886	0.000011	0.000010
0.06	820.345443	820.345442	820.345446	0.000003	0.000004
0.07	822.938545	822.938533	822.938592	0.000046	0.000059
0.08	825.290479	825.290428	825.290624	0.000145	0.000197
0.09	827.396781	827.396618	827.397148	0.000367	0.000530
0.10	829.253210	829.252774	829.254111	0.000902	0.001337

Total Absolute Error (vs RK45): ADM = 0.001496, HPM = 0.002158

CONCLUSION: For L, ADM provides a closer approximation.

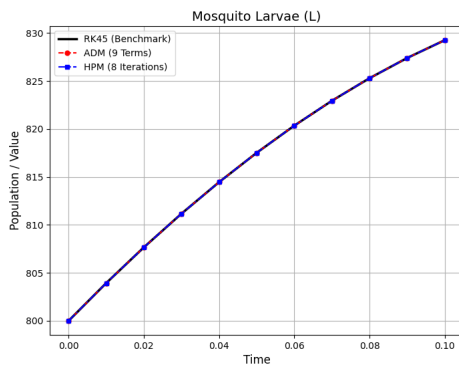


Figure 13: Graph of the Mosquito Larvae Population showing the ADM, HPM, and RK45

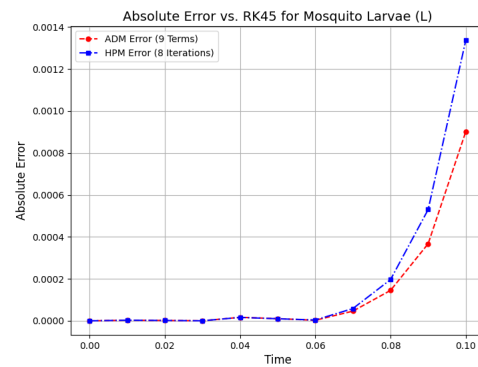


Figure 14: ADM and HPM error plots for the Mosquito Larvae Population vs RK45

Table 12: Comparison for Mosquito Pupae (P)

Time	ADM_Value	HPM_Value	RK45_Value	ADM_Error	HPM_Error
0.00	400.000000	400.000000	400.000000	0.000000	0.000000
0.01	404.136572	404.136572	404.136574	0.000002	0.000002
0.02	408.134311	408.134311	408.134312	0.000001	0.000001
0.03	411.989348	411.989348	411.989348	0.000000	0.000000
0.04	415.697880	415.697880	415.697888	0.000009	0.000008
0.05	419.256199	419.256200	419.256205	0.000005	0.000005
0.06	422.660726	422.660724	422.660726	0.000000	0.000002
0.07	425.908023	425.908011	425.908041	0.000018	0.000030
0.08	428.994814	428.994769	428.994869	0.000055	0.000100
0.09	431.917992	431.917856	431.918126	0.000134	0.000271
0.10	434.674618	434.674258	434.674943	0.000325	0.000685
Total Absolute Error (vs RK45): ADM = 0.000550, HPM = 0.001104					
CONCLUSION: For P, ADM provides a closer approximation.					

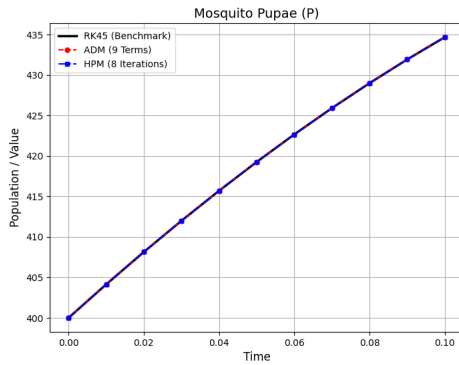


Figure 15: Graph of the Mosquito Pupae Population showing the ADM, HPM, and RK45

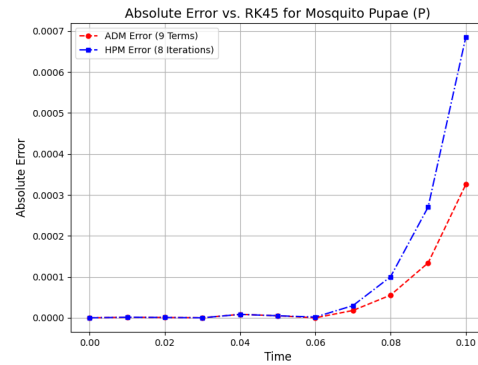


Figure 16: ADM and HPM error plots for the Mosquito Pupae Population vs RK45

Table 13: Comparison for Susceptible Mosquitoes (S_m)

Time	ADM_Value	HPM_Value	RK45_Value	ADM_Error	HPM_Error
0.00	300.000000	300.000000	300.000000	0.000000	0.000000
0.01	299.774695	299.774695	299.774669	0.000026	0.000026
0.02	299.435823	299.435825	299.435809	0.000014	0.000016
0.03	298.974001	298.974010	298.974010	0.000009	0.000001
0.04	298.382210	298.382239	298.382103	0.000106	0.000136
0.05	297.655307	297.655374	297.655287	0.000020	0.000087
0.06	296.789680	296.789800	296.789764	0.000084	0.000036
0.07	295.783045	295.783214	295.782717	0.000328	0.000497
0.08	294.634418	294.634560	294.632883	0.001536	0.001677
0.09	293.344288	293.344151	293.339640	0.004648	0.004511
0.10	291.915064	291.914012	291.902740	0.012324	0.011272
Total Absolute Error (vs RK45): ADM = 0.019096, HPM = 0.018260					
CONCLUSION: For S_m , HPM provides a closer approximation.					

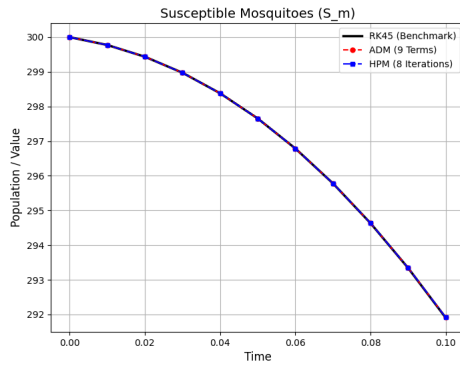


Figure 17: Graph of the Susceptible Mosquito Population showing the ADM, HPM, and RK45

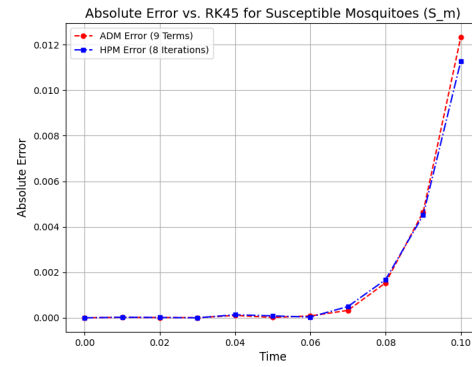


Figure 18: ADM and HPM error plots for the Susceptible Mosquito Population vs RK45

6.1. Discussion

6.1.1. General Observation

From the plotted figures above, we can see a reasonable explanation of the dynamics of the Malaria. There's a decrease in the number of unaware susceptible individuals and an increase in the number of aware susceptible individuals provided there's an increase in the level of awareness and the population. For this small-time interval, we are able to see this effect performed by the two semi-analytical methods. The number of infected mosquitoes and the number of susceptible mosquito populations also follows the same dynamics of reduction as the awareness campaigns increases showing an effectiveness of the awareness campaigns for this small-time span. The number of infected individuals surges in this small-time span, this may be due to the number of exposed individuals already in the population, since it is visible that the number of exposed individuals also increases for this small-time span. However, if the awareness campaigns are maintained well, we should see a decrease in the number of infected humans over a long-time span.

Table 14: Comparison for Exposed Mosquitoes (E_m)

Time	ADM_Value	HPM_Value	RK45_Value	ADM_Error	HPM_Error
0.00	30.000000	30.000000	30.000000	0.000000	0.000000
0.01	31.131114	31.131114	31.131141	0.000027	0.000027
0.02	32.263887	32.263884	32.263901	0.000015	0.000017
0.03	33.406764	33.406754	33.406755	0.000009	0.000001
0.04	34.565879	34.565849	34.565992	0.000113	0.000143
0.05	35.745563	35.745496	35.745588	0.000025	0.000092
0.06	36.948714	36.948594	36.948632	0.000083	0.000038
0.07	38.177011	38.176838	38.177360	0.000349	0.000522
0.08	39.430948	39.430787	39.432548	0.001601	0.001762
0.09	40.709663	40.709740	40.714477	0.004814	0.004738
0.10	42.010476	42.011366	42.023208	0.012732	0.011842
Total Absolute Error (vs RK45): ADM = 0.019768, HPM = 0.019180					
CONCLUSION: For E_m , HPM provides a closer approximation.					

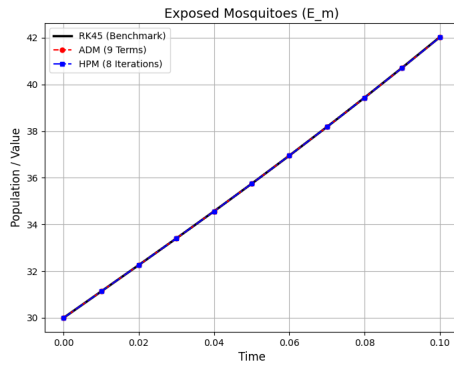


Figure 19: Graph of the Exposed Mosquito Population showing the ADM, HPM, and RK45

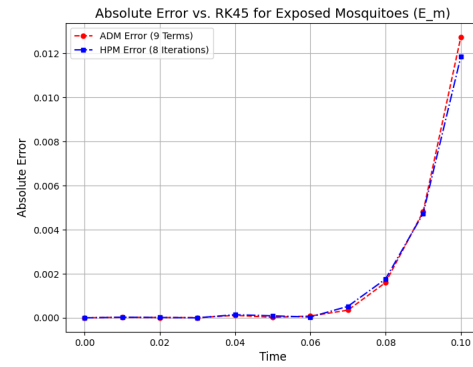


Figure 20: ADM and HPM error plots for the Exposed Mosquito Population vs RK45

Table 15: Comparison for Infected Mosquitoes (I_m)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	50.000000	50.000000	50.000000	0.000000	0.000000
0.01	50.004104	50.004104	50.004104	0.000000	0.000000
0.02	49.990600	49.990600	49.990600	0.000000	0.000000
0.03	49.959504	49.959504	49.959504	0.000000	0.000000
0.04	49.910844	49.910844	49.910844	0.000001	0.000001
0.05	49.844668	49.844668	49.844668	0.000000	0.000000
0.06	49.761041	49.761041	49.761041	0.000000	0.000000
0.07	49.660049	49.660049	49.660051	0.000002	0.000002
0.08	49.541798	49.541795	49.541803	0.000006	0.000008
0.09	49.406412	49.406405	49.406426	0.000014	0.000021
0.10	49.254035	49.254017	49.254071	0.000036	0.000054

Total Absolute Error (vs RK45): ADM = 0.000059, HPM = 0.000087

CONCLUSION: For I_m , ADM provides a closer approximation.

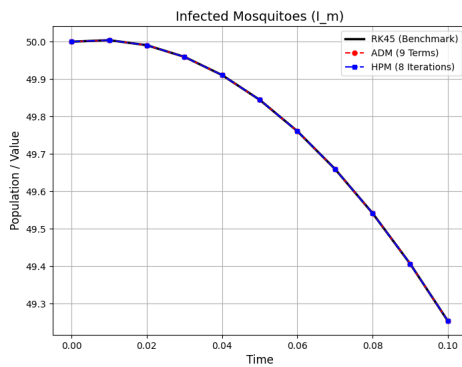


Figure 21: Graph of the Infected Mosquito Population showing the ADM, HPM, and RK45

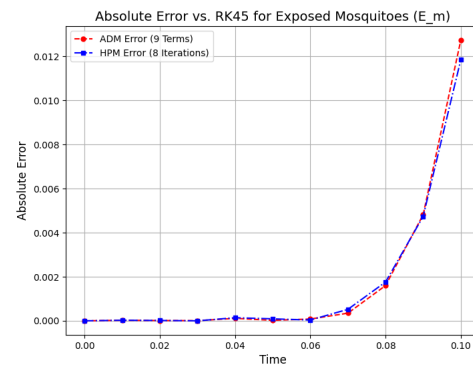


Figure 22: ADM and HPM error plots for the Infected Mosquito Population vs RK45

6.1.2. *HPM versus RK45: Superior Initial Accuracy with Progressive Divergence*

The Homotopy Perturbation Method demonstrates remarkable precision in the initial phase of simulation, particularly for human population compartments where it closely tracks the benchmark RK45 solution. This alignment is mathematically evidenced by significantly smaller absolute errors in early time steps ($t < 0.04$), suggesting that HPM's perturbation series expansion provides an excellent approximation to the true solution during the initial transient phase. However, as the simulation progresses beyond $t = 0.07$, the method exhibits progressive divergence from the RK45 solution, with error accumulation becoming particularly pronounced in compartments experiencing rapid exponential growth. This behavior indicates that while HPM captures the initial dynamics effectively, its perturbation approach may struggle to maintain accuracy as nonlinear effects become more dominant over time.

6.1.3. *ADM versus RK45: Enhanced Long-Term Stability and Predictable Error Growth*

The Adomian Decomposition Method displays superior long-term stability characteristics, maintaining consistent alignment with the RK45 solution across extended simulation periods. Mathematically, this is reflected in more gradual and predictable error growth patterns, with ADM demonstrating particular strength in handling the complex dynamics of mosquito life cycle compartments. The method's decomposition approach appears well-suited for capturing the underlying system structure, as evidenced by its exceptional performance in compartments exhibiting exponential growth patterns. Furthermore, ADM maintains competitive accuracy throughout the entire temporal domain for awareness programs and infected mosquito populations, suggesting its decomposition strategy provides a more robust framework for handling the system's nonlinearities over extended time horizons. We conclude finally that the HPM provides better approximation for: Susceptible Humans, Aware Susceptible Humans, Infected Humans, Recovered Humans, Susceptible Mosquitoes, Exposed Mosquitoes while the ADM provides better approximation for: Exposed Humans, Mosquito Eggs, Mosquito Larvae, Mosquito Pupae, Infected Mosquitoes, Awareness Programs.

Overall, ADM shows superior performance for mosquito-related compartments, while HPM performs better for most human compartments.

Table 16: Comparison for Awareness Programs (A)

Time	ADM Value	HPM Value	RK45 Value	ADM Error	HPM Error
0.00	0.000000	0.000000	0.000000	0.000000	0.000000
0.01	0.082963	0.082963	0.082963	0.000000	0.000000
0.02	0.165945	0.165945	0.165945	0.000000	0.000000
0.03	0.248998	0.248998	0.248998	0.000000	0.000000
0.04	0.332152	0.332152	0.332153	0.000001	0.000001
0.05	0.415427	0.415426	0.415427	0.000000	0.000001
0.06	0.498827	0.498826	0.498826	0.000001	0.000000
0.07	0.582351	0.582348	0.582352	0.000002	0.000004
0.08	0.665987	0.665981	0.665996	0.000009	0.000015
0.09	0.749715	0.749703	0.749743	0.000028	0.000040
0.10	0.833504	0.833480	0.833580	0.000076	0.000099
Total Absolute Error (vs RK45): ADM = 0.000117, HPM = 0.000161					
CONCLUSION: For A, ADM provides a closer approximation.					

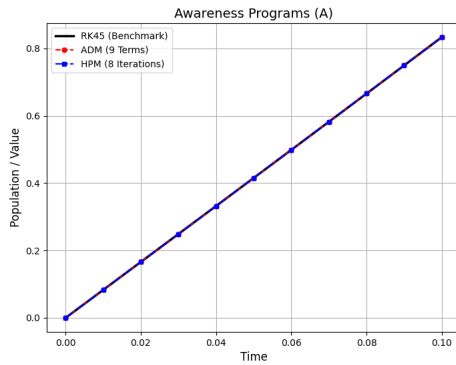


Figure 23: Graph of the Awareness Program Population showing the ADM, HPM, and RK45

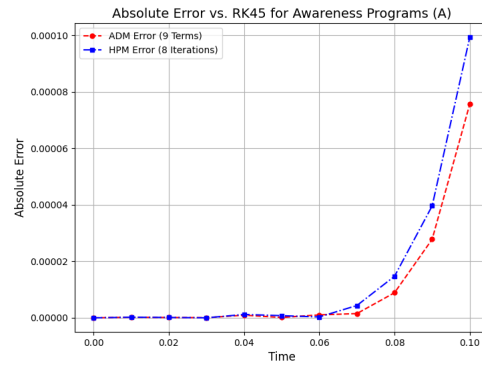


Figure 24: ADM and HPM error plots for the Awareness Program Population vs RK45

6.1.4. ADM versus HPM: Error Analysis

The error plots (Figures 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24) visualize the absolute difference between the semi-analytical solutions (ADM and HPM) and the high-accuracy numerical solution (RK45). A lower error indicates a better approximation. The key takeaway is that neither ADM nor HPM is universally superior. The patterns observed can be grouped into three regimes:

Regime 1: HPM Demonstrates Clear Superiority

Figure 4 shows that the error plot for HPM is almost negligible and remains very close to zero, while ADM's error grows noticeably over time. This suggests HPM's perturbation approach is exceptionally well-suited for the dynamics of this compartment, which is influenced by awareness campaigns and transitions from the unaware state.

Regime 2: ADM Demonstrates Clear Superiority

Figures 7, 9, and 11 show the most consistent trend for this scenario. For all three immature mosquito stages, the error for ADM is significantly lower and grows more slowly than for HPM. The dynamics of these compartments are governed by logistic growth terms,

which ADM's polynomial decomposition seems to handle more effectively.

Regime 3: Competitive or Mixed Performance

Figure 6 shows a strongly mixed case. ADM starts with a much lower error, but its error grows rapidly after $t = 0.07$. HPM starts with a higher error but remains more stable, leading to a crossover point where HPM becomes the more accurate approximation later in the interval. This suggests ADM captures the initial dynamics well but may diverge faster.

6.2. Effects of Seasonality and Awareness Campaigns

We applied the ADM method to show the effect of seasonality and awareness on the transmission dynamics of malaria using the model equations and parameter values by Ochieng [33]. We decided to first show the effect of the awareness campaign on some compartments in the malaria dynamics.

Using the parameter values provided by Ochieng [33], we were able to obtain the following trends for the following compartments plotted against time for a small amount of time. However, we increased the awareness campaign parameters δ and ω by a factor of 5 and 10 to indicate the level of Effectiveness of the awareness campaign on four compartments in the dynamics. This change to show the effectiveness of the campaign was compared with the baseline values given by Ochieng [33]. Based on the plot obtained for the infected human compartment, we saw a sharp decline in the number of unaware susceptible individuals as the awareness parameters were varied by a factor of 5 and a factor of 10. This shows the effectiveness of the awareness campaign on the reduction of infected individuals in the population.

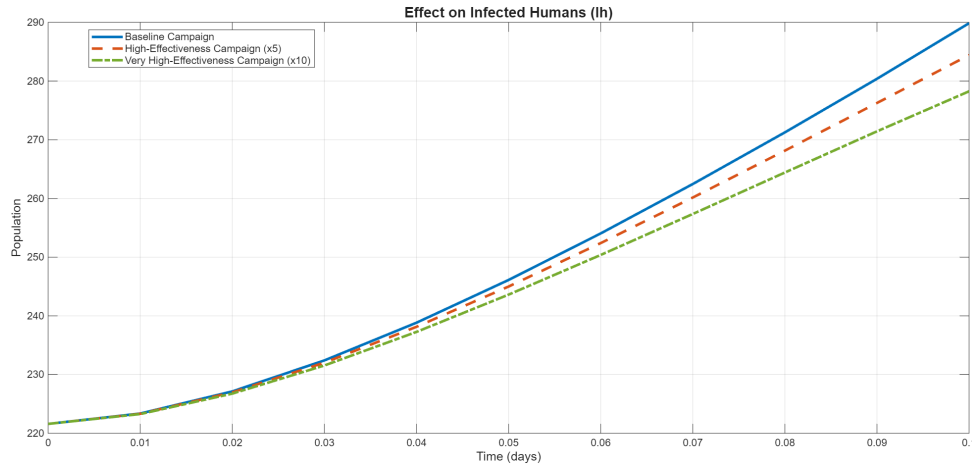


Figure 25: Graph of the effect of the awareness campaign on infected compartment on varying δ and ω using the ADM method.

We later performed a sensitivity analysis to show the effect of seasonal variation on the transmission of mosquitoes in the population. We decided to vary the recruitment rate of mosquitoes b_m by considering a 50% increase and a 50% decrease in the value of b_m for the wet season and dry season, respectively, while the normal season was the value for the parameter given in [33]. The Figure for the Mosquito Egg population is given below:

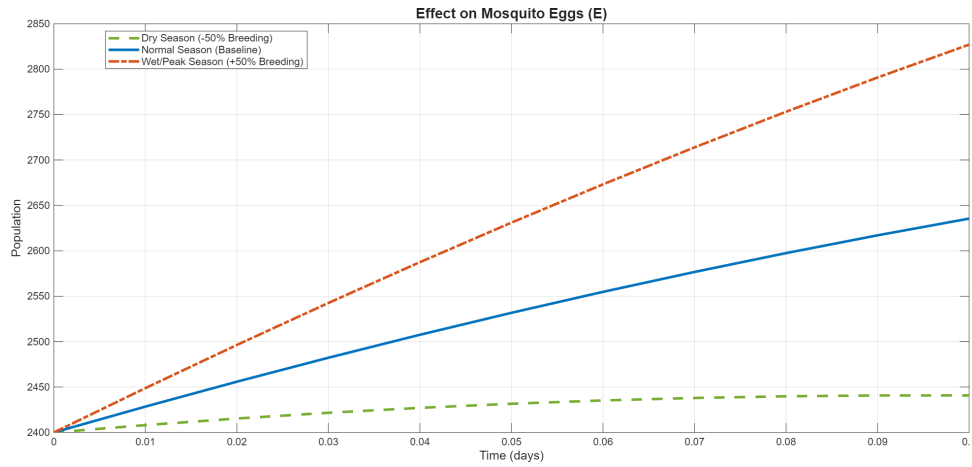


Figure 26: Graph of the effect of seasonal variation on the Mosquito Eggs Population using the ADM method

You can see that a 50% increase during the wet season leads to more breeding sites for mosquitoes, while a 50% decrease during the dry season results in a decrease in the number of breeding sites for mosquitoes.

Additionally, we decided to see the effect of certain mosquito compartments when compared with the effectiveness of the awareness campaign when the awareness campaign parameters are increased by a factor of 5 and 10. The plots below show a decrease in the number of infected mosquitoes when the different scenarios of the awareness campaign were carried out. Telling us that the more effective our campaign is, the fewer the number of infected mosquitoes. This is because people will utilize the material given to them during the campaign to reduce their contact with susceptible mosquitoes or even infected mosquitoes. The number of susceptible mosquitoes also decreases as the awareness campaign becomes very effective.

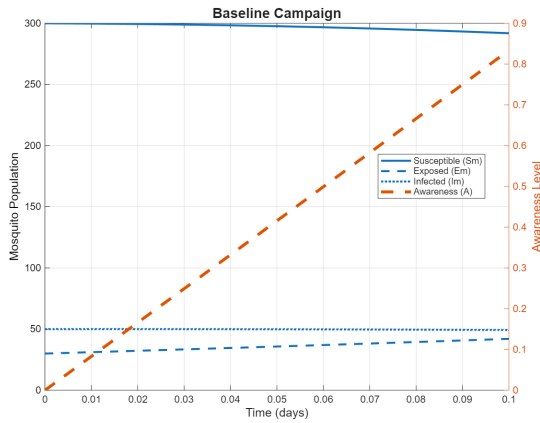


Figure 27: Graph of the effect of the baseline campaign using the ADM method

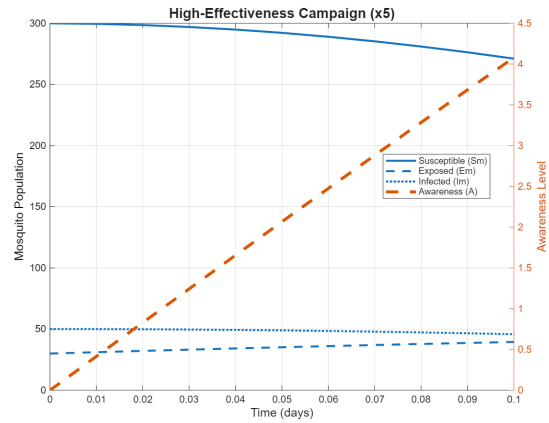


Figure 28: Graph of the effect of the high-effectiveness campaign using the ADM method

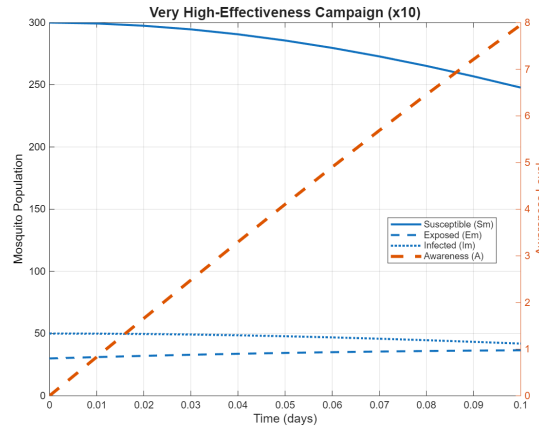


Figure 29: Graph of the effect of the very high-effectiveness campaign using the ADM method

7. Conclusion

This study successfully applied and critically compared the performance of two semi-analytical methods, the Adomian Decomposition Method (ADM) and the Homotopy Perturbation Method (HPM), to a high-dimensional, non-linear, and seasonally-forced malaria transmission model. The model's complexity, featuring twelve compartments that integrate mosquito life stages, human awareness, and seasonal forcing, provided a rigorous test case for evaluating the efficacy of these methods against a numerical benchmark (RK45).

The central finding of this comparative analysis is that neither method is universally superior; instead, their performance is compartment-specific. The Homotopy Perturbation Method (HPM) demonstrated remarkable precision for most human population dynamics—specifically for susceptible, infected, and recovered humans—particularly in the initial phase of the simulation. This suggests that HPM's perturbation structure is exceptionally well-suited for capturing the transient dynamics of human disease compartments.

In contrast, the Adomian Decomposition Method (ADM) exhibited superior long-term stability and consistently lower errors for the mosquito life-cycle stages (eggs, larvae, pupae) and the awareness program variable. ADM's systematic decomposition of nonlinearities appears to provide a more robust framework for handling the logistic growth and decay terms inherent in the mosquito population dynamics.

Furthermore, the application of ADM to analyze the effects of control measures yielded critical public health insights. The sensitivity analysis confirmed that intensifying awareness campaigns leads to a substantial reduction in both infected human and mosquito populations, underscoring the vital role of public education and insecticide use in malaria control. Similarly, varying the mosquito recruitment rate to simulate seasonal effects highlighted how environmental conditions directly influence mosquito abundance and, consequently, disease transmission potential.

In conclusion, this work establishes that both ADM and HPM are powerful and viable tools for obtaining semi-analytical insights into complex epidemiological systems. The choice between them should be guided by the specific focus of the analysis: HPM is preferable for short-term human population dynamics, while ADM offers greater reliability for systems involving resource-limited growth, such as mosquito populations, and for extended time horizons. Future work could explore the hybridization of these methods or their integration with optimization algorithms for parameter estimation and optimal control strategy development, further bridging the gap between mathematical theory and practical disease management.

Declarations

Competing interests

The authors declare that there is no competing interests.

Author's contributions

Abimbola Abolarinwa: Conceptualization, Methodology, Analysis, Writing, Reviewing and Editing. Isaac Olusola: Investigation, Simulation, Validation, Analysis, Writing and Editing. All authors contributed equally to the writing and editing of this manuscript.

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