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A Fluctuation Analysis of United State Dollar Price in the Philippines: A Stochastic White Noise Approach

Rechie S. Magsalay [a,*]

[a] Jose Rizal Memorial State University, Dapitan City 7101, Philippines

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Abstract

Given the fluctuating trend in the United States Dollar (USD) exchange rate in the Philippines, a stochastic analysis employing a white noise framework was conducted to examine the daily USD price. This approach was utilized to characterize the statistical behavior of the exchange rate by computing the Mean Square Deviation (MSD) based on 11,205 daily data points spanning from January 1978 to July 2023. The resulting theoretical MSD was used to generate a Probability Density Function (PDF), which effectively describes the empirical distribution of the USD price fluctuations in the Philippine context.

Keywords: United State Dollar, Fluctuation, Memory Function, Mean Square Deviation, and Probability Density Function

1. Introduction

The US Dollar, as the world currency, drives economic growth in some countries through international trade [1],[2] This currency serves as the global medium for world trade markets and has significant impacts on various countries, particularly the Philippines. In recent years, the abrupt increase in the USD exchange rate in the Philippines has been noted by the Bangko Sentral ng Pilipinas (BSP). In 2007, the Philippine economy grew rapidly, emerging as one of the top-performing countries in Asia, with the USD hitting as high as P41/USD [3]. Currently, the value of the USD in pesos is approximately or near P60/USD, which encourages many Filipinos to work abroad or to engage online with international clients.

The increase in the number of Filipinos working abroad and remotely has significantly impacted the Philippine economy. The recent rise in the USD exchange rate, approaching P60/USD, has led to several noteworthy developments [4]. Remittances, trade, investment, tourism, economic stability, and debt financing are just a few of the effects on the Philippine economy. These impacts result in crucial changes and influences, affecting household incomes and government policies related to trade and investment [5],[6].

*Corresponding author: rechie.magsalay@jrmsu.edu.ph

On the other hand, several studies have examined the exchange rate of the USD in the Philippines. One study investigated factors affecting the real exchange rate from 1973 to 2014, including gross domestic product, money supply, net foreign assets, budget deficits, import restrictions, and oil prices [7]. Another study aimed to determine the effects of interest rates, inflation rates, and government expenditure on the USD exchange rate [8].

Furthermore, some papers have attempted to forecast USD trends in the Philippines, such as the study by Nhoriel I. Toledo, which aimed to improve forecasts for the USD exchange rate using Autoregressive Integrated Moving Average (ARIMA) modeling [9]. Similarly, Urrutia et al. provided models and forecasts for the behavior of the USD exchange rate in the Philippines using time series analysis [10].

In this paper, we consider the fluctuation of the USD price in the Philippines as a random variable and will utilize a stochastic analysis with a white noise approach. The main goal of this paper is to provide mathematical models that describe the behavior of the USD price in the Philippines.

2. Methodology

This study will utilize the daily price of the USD (in PHP) raw data retrieved from the official website of the Bangko Sentral ng Pilipinas [11]. The dataset contains 11,205 daily price points of the USD in PHP, covering the period from January 1978 to July 2023. The collected data will be analyzed using the method of white noise analysis for Brownian motion with a memory function. To manipulate the data, we will apply the following steps [12],[13]:

1. Plot the raw data of the daily USD price (in PHP) using PYTHON programming language.
2. Determine the best linear fitting of the daily USD price (in PHP).
3. Compute the fluctuations of the data which are the differences between the data points and the points from the best linear fit.
4. Compute the Empirical Mean Square Deviation (MSD).
5. Plot the points (log (time), log (MSD)). The graph generated by these points will be the basis to find the best fitted Theoretical MSD.
6. From the Theoretical MSD, the memory function will be derived to find the Probability Density Function (PDF) of the daily price of USD (in PHP).

A stochastic random variable $x(T)$ with no memory of its past can be written as [14],

$$x(T) = x_0 + \sqrt{2D}B(T) \tag{2.1}$$

where $B(T)$ is the ordinary Brownian motion x_0 and is the starting position. To pin down the endpoint at $x(T) = x_T$ at time t, we consider the delta constant, which is written as $\delta(x(T) - x_T)$ The probability density function can be express as the integral of the delta function over the Gaussian white noise measure $d_\mu(\omega)$.

More precisely, the probability density function is written as,

$$P(x_T, T; x_0, 0) = \int \delta(x(T) - x_T) d_\mu(\omega) \tag{2.2}$$

However, several phenomena do not fit the randomness of the Brownian motion in equation 2.1. Thus, to extend this application in real-world phenomena, a memory function $f(T-t)$ is added to allow the path $x(T)$ to have a memory of its past. To parametrize the path, the path will now be written as

$$x(T) = x_0 + \int_0^T f(T-t)h(t)\omega(t)dt \tag{2.3}$$

Using equation 2.2, the probability density function can now be written as

$$P(x_T, T; x_0, 0) = \int \delta\left(x_0 + \int_0^T f(T-t)h(t)\omega(t)dt - x_T\right) d_\mu(\omega) \tag{2.4}$$

By applying the Fourier representation of the delta function, we can rewrite the probability density function from equation 2.4 as

$$P(x_T, T; x_0, 0) = \frac{1}{2\pi}\int e^{ikg(T)\int_0^T f(T-t)h(t)\omega(t)dt} d_\mu(\omega) \int_{-\infty}^{+\infty} e^{ik(x_0-x_T)} dk \tag{2.5}$$

Now, taking advantage of the characteristic functional for the $d_\mu(\omega)$.

$$C(\xi) = \int e^{i\int_0^T \omega(t)\xi(t)dt} d_\mu(\omega) = e^{-\frac{1}{2}\int_0^T \xi^2 dt} \tag{2.6}$$

so, the probability density function can be expressed as

$$P(x_T, T; x_0, 0) = \frac{1}{2\pi} e^{-\frac{1}{2}k^2 g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt} \int_{-\infty}^{+\infty} e^{ik(x_0-x_T)} dk \tag{2.7}$$

Simplifying further the equation 2.7, the final expression for the probability density function is

$$P(x_T, T; x_0, 0) = \frac{1}{\sqrt{2\pi g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt}} \exp\left(\frac{-(x_0-x_T)^2}{2g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt}\right) \tag{2.8}$$

where $f(T-t)$ and $h(t)$ are the memory function and $g(T)$ is the regulating function for fluctuation. The MSD, which represent the deviation from the mean, can be express as [14],

$$MSD = < x^2 > - < x >^2 \tag{2.9}$$

Evaluating the mean square deviation, we need to solve the first and second moment: $< x >$ and $< x^2 >$. To do this, we must calculate the following

$$< x > = \int_{-\infty}^{+\infty} x P(x_T, T; x_0, 0) dx \tag{2.10}$$

and,

$$< x^2 >= \int_{-\infty}^{+\infty} x^2 P(x_T, T; x_0, 0) dx \tag{2.11}$$

The first moment $< x >$, equation 2.10, is evaluated as

$$< x >= \int_{-\infty}^{+\infty} x P(x_T, T; x_0, 0) dx = \int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt}}$$

$$\exp\left(\frac{-(x_0 - x_T)^2}{2g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt} dx \right)$$

$$< x >= x_0 \tag{2.12}$$

and for the second moment, equation 2.11,

$$< x^2 >= \int_{-\infty}^{+\infty} x^2 P(x_T, T; x_0, 0) dx = \int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt}}$$

$$\exp\left(\frac{-(x_0 - x_T)^2}{2g(T)^2 \int_0^T [f(T-t)h(t)]^2 dt} dx \right)$$

$$< x^2 >= x_0^2 + [g(T)]^2 \int_0^T [f(T-t)h(t)]^2 dt \tag{2.13}$$

Thus, equation 2.9 yields,

$$\mathrm{MSD} = [g(T)]^2 \int_0^T [f(T-t)h(t)]^2 dt \tag{2.14}$$

Using equation 2.14, the probability density function from equation 2.8 may be written as

$$P(x_T, T; x_0, 0) = \frac{1}{\sqrt{2\pi \mathrm{MSD}}} \exp\left(\frac{-(x_0 - x_T)^2}{2\mathrm{MSD}} \right) \tag{2.15}$$

3. Results and Discussions

This section presents the entire process of white noise analysis conducted for the daily price of the USD from January 1978 to July 2023 in the Philippines. The analysis was performed using an Excel file and implemented in PYTHON Programming Language, utilizing the *NumPy*, *Matplotlib*, and *SciPy* modules within a Jupyter Notebook.

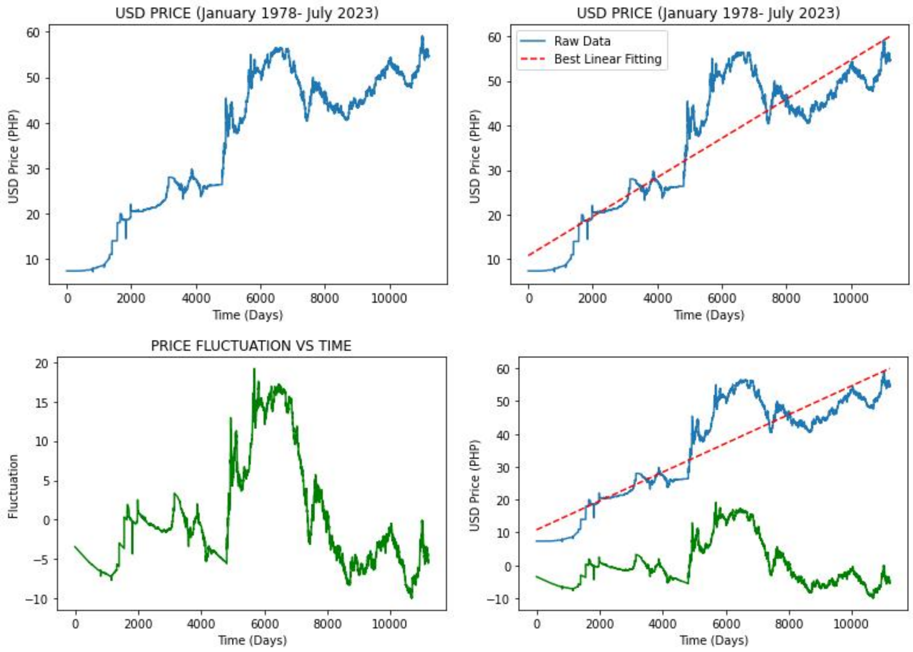

Figure 1: Graphical Representation of the Data: (a) USD Price (January 1978 - July 2023), (b) Best Linear Fitting of the Data (red broken line), (c) Fluctuation of the USD Price Data, and (d) The combination graph representation of (b) and (c).

3.1. Empirical Data

Figure 1 represents the graphs of the daily price of the USD in Philippine Pesos. The dataset was downloaded from the open-access economic domain of the Bangko Sentral ng Pilipinas [11]. It contains the daily price of the USD (in PHP) from January 1978 to July 2023, comprising 11,205 data points. The best linear fit was generated and determined to be **f(t)** = 0.0044**t** + 10.8179, where **f(t)** represents the price of the USD in Philippine Pesos at a given time **t** in days. Additionally, the fluctuations in the USD price over time were calculated by subtracting the original dataset from **f(t)**.

The empirical Mean Square Deviation (MSD) of the fluctuations was also calculated, as shown in Figure 2. This was obtained by taking the differences between two positions separated by time, squaring the difference, and moving to the next position until all data points were covered. The squares were then summed and averaged. Additionally, to represent the empirical MSD, a log-log plot (on the right) and an algebraic transformation of (Time,MSD)$\to$(log(Time),log(MSD)) (on the left) were generated to provide clearer and smoother graphs for the empirical data. Despite the differences in axis scales, the shape of the MSD graphs remains consistent across different scales.

3.2. Theoretical MSD

Using the 16 memory functions provided by C.C. Bernido and M.V.C. Bernido [14], we derived the theoretical Mean Square Deviation (MSD) that fitted the empirical MSD. A trial-and-error approach was employed, along with adjustments of certain parameters. Figure 3 illustrates how the theoretical MSD was fitted to the empirical MSD.

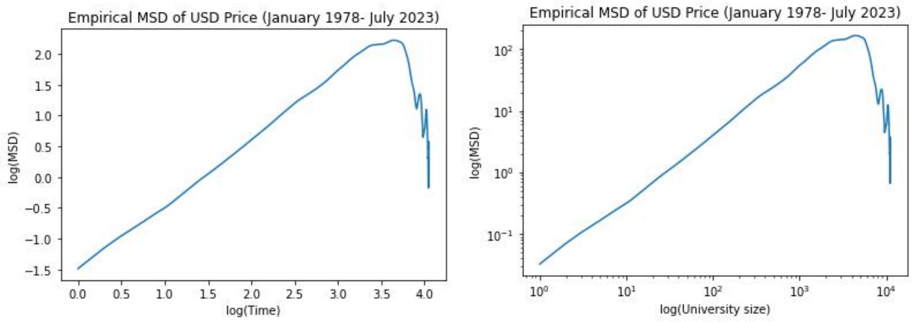

Figure 2: Empirical MSD

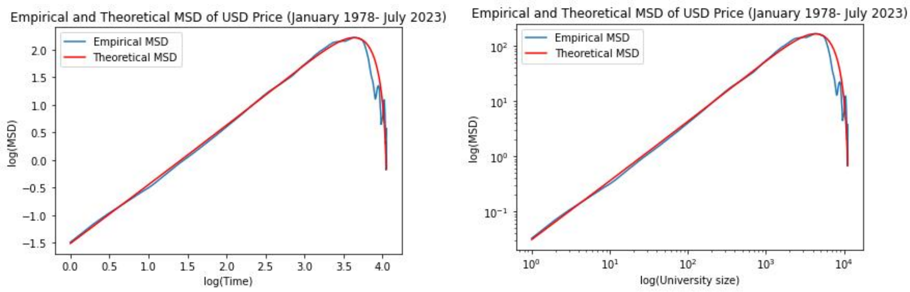

Figure 3: Theoretical MSD

To get the fitted theoretical MSD, the memory function,

$$f = (T - t)^{\frac{\mu - 1}{2}}$$

and,

$$h = \frac{\sqrt{\cos(at)}}{t^{\frac{1-\mu}{2}}},$$

where chosen, and the regulating function was also adjusted as,

$$g(T) = \sqrt{J_{\mu - \frac{1}{2}}(\frac{aT}{2})\cos^{1.5}(\frac{aT}{2})e^{(\frac{T}{100000})^{0.3}}},$$

so the Theoretical MSD is,

$$MSD = [g(T)]^2 \int_0^T [f(T - t)h(t)]^2 dt$$

$$MSD = \left[\sqrt{J_{\mu - \frac{1}{2}}\left(\frac{aT}{2}\right)\cos^{1.5}\left(\frac{aT}{2}\right)e^{\left(\frac{T}{100000}\right)^{0.3}}}\right]^2 \int_0^T \left[(T - t)^{\frac{\mu - 1}{2}}\frac{\sqrt{\cos(at)}}{t^{\frac{1-\mu}{2}}}\right]^2 dt \qquad (3.1)$$

Simplifying further, this leads to,

$$MSD = \left[J_{\mu-\frac{1}{2}} \left(\frac{aT}{2} \right) \cos^{1.5} \left(\frac{aT}{2} \right) e^{\left(\frac{T}{100000}\right)^{0.3}} \right] \left[N \frac{\cos\left(\frac{aT}{2}\right) \Gamma(\mu) J_{\mu-\frac{1}{2}}\left(\frac{aT}{2}\right)}{\pi^{-\frac{1}{2}} T^{\frac{1}{2}-\mu} a^{\mu-\frac{1}{2}}} \right] \tag{3.2}$$

or,

$$MSD = N e^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu) \pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right) \left(J_{\mu-\frac{1}{2}} \left(\frac{aT}{2}\right)\right)^2 \tag{3.3}$$

where N is the normalization constant, Γ is a gamma function, and $J_{\mu-\frac{1}{2}}$ is a Bessel's function.

To have a better fit of empirical MSD, the parameters of the theoretical plot was done by trial and error. The parameters of the theoretical plot were $N = 0.57$, $a = 0.0002658$, and $\mu = 0.85$. These parameters have vital roles on fitting the empirical MSD; (i) the normalization constant, the parameter N, was used to move the graph up and down, (ii) the parameter a , was used to stretch the graph left and right, and (iii) the parameter μ, was used to tilt the plot. Lastly, the parameters N, a, and μ, can be related to some factors of the continues increase of the price of USD in the Philippines.

3.3. Probability Density Function

The probability density function was obtained using the following expression,

$$P(x_T, T; x_0, 0) = \frac{1}{\sqrt{2\pi MSD}} exp\left(\frac{-(x_0 - x_T)^2}{2MSD} \right) \tag{3.4}$$

and the Theoretical MSD we obtained is,

$$MSD = N e^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu) \pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right) \left(J_{\mu-\frac{1}{2}} \left(\frac{aT}{2}\right)\right)^2 \tag{3.5}$$

Therefore, the exact solution of the probability density function is,

$$P(x_T, T; x_0, 0) = \frac{1}{\sqrt{2\pi N e^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu) \pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right) \left(J_{\mu-\frac{1}{2}} \left(\frac{aT}{2}\right)\right)^2}} exp\left(\frac{-(x_0 - x_T)^2}{2N e^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu) \pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right) \left(J_{\mu-\frac{1}{2}} \left(\frac{aT}{2}\right)\right)^2} \right), \tag{3.6}$$

where $N = 0.57$, $a = 0.0002658$, and $\mu = 0.85$.

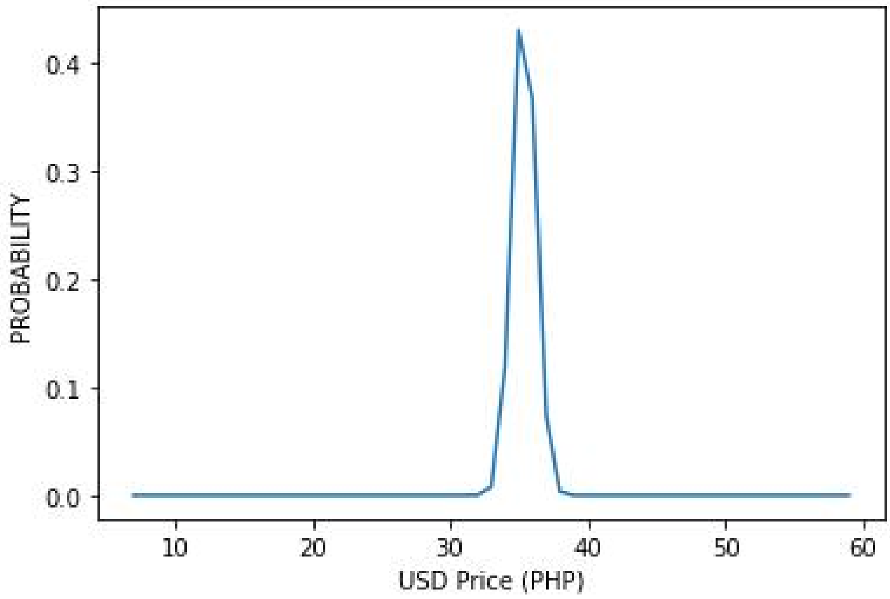

Figure 4: Probability Density Function with $x_0 = PHP35.39perUSD$ in equation (3.6)

Figure 4 shows a model that describes the behavior of the USD price at time $t = 11205$ days with $x_0 = PHP35.39$, the average of the daily price of USD. This kind of model describing the data was also used to the paper of Maulion M., et.al. [13]. Another model description, based on the paper of Barredo W., et.al. [15], was also shown in Figure 5, it shows the PDF over time of the comparison between the empirical data and the theoretical data (equation 21) with $(x_0 - x_T) = PHP35.39$. Using the same parameters to the theoretical MSD, the trends and behavior of the theoretical path follows the empirical data but not smoothly on the days: $2000^{th} - 4000^{th}$ day and $6000^{th} - 8000^{th}$ day.

4. Conclusions and Recommendations

In this paper, the ultimate goal is to describe the behavior of the daily price of the USD in PHP using mathematical models and graphs. The researcher employed stochastic analysis with a white noise approach, treating the evolution of the daily USD price over time as a random variable. By analyzing the fluctuating data of the USD price, the empirical Mean Square Deviation (MSD) was compared with the theoretical MSD derived from the selected memory functions as follows:

$$f = (T - t)^{\frac{\mu - 1}{2}} \text{ and, } h = \frac{\sqrt{\cos(at)}}{t^{\frac{1-\mu}{2}}}$$

With that, the theoretical MSD was expressed as,

$$MSD = Ne^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu) \pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right) \left(J_{\mu-\frac{1}{2}}\left(\frac{aT}{2}\right)\right)^2$$

and also, the PDF was described as,

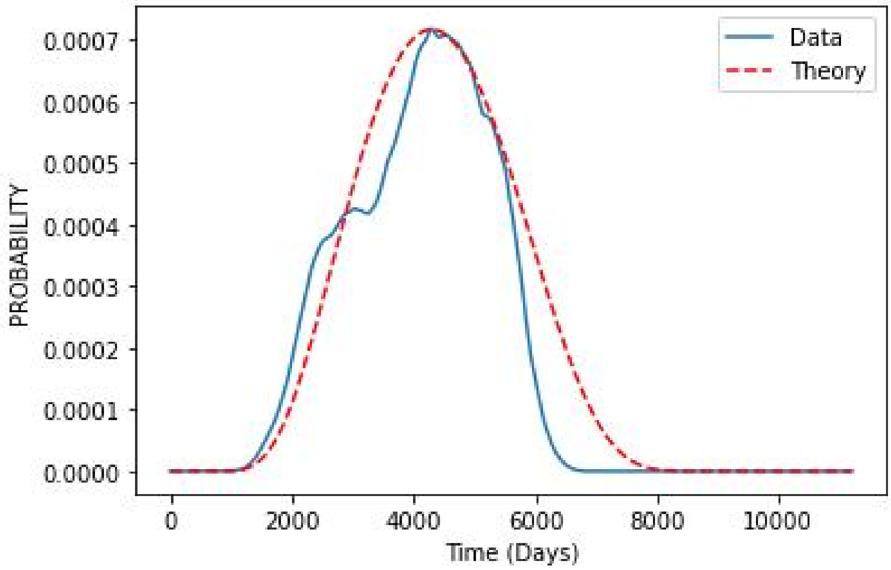

Figure 5: Probability Density Function with $(x_0 - x_T) = \text{PHP}35.39perUSD$ in equation (3.6)

$$P(x_T, T; x_0, 0) = \frac{1}{\sqrt{2\pi N e^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu)\pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right)\left(J_{\mu-\frac{1}{2}}\left(\frac{aT}{2}\right)\right)^2}}$$

$$\exp\left(\frac{-(x_0 - x_T)^2}{2N e^{\left(\frac{T}{100000}\right)^{0.3}} \Gamma(\mu)\pi^{\frac{1}{2}} T^{-\frac{1}{2}+\mu} a^{-\mu+\frac{1}{2}} \cos^{2.5}\left(\frac{aT}{2}\right)\left(J_{\mu-\frac{1}{2}}\left(\frac{aT}{2}\right)\right)^2}\right)$$

where $N = 0.57$, $a = 0.0002658$, and $\mu = 0.85$. With the PDF we obtained, it provides and uses two models [13],[15] to describe the behavior of the price of the USD in the Philippines.

For further development of this study, it is recommended to provide additional mathematical models to describe the behavior of the USD price. Additionally, determining the diffusion coefficient [12],[13] of the data fluctuations is suggested to better predict the long-term behavior of the USD price in the Philippines.

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