Journal of Mathematical Analysis and Modeling
jmam.sabapub.com
ISSN 2709-5924

J Math Anal & Model
(2025)6(2) : 43-59
doi:10.48185/jmam.v6i2.1215

Modeling the Dynamics of Corruption and Optimal Control in Public Sectors

Odeli J. Kigodi [a,*], Mohamedi S. Manjenga [b], Fadhili M. Mrope [c], Honda N. Masasila [d]

[a] Department of Mathematics and Statistics, College of Science and Technical Education, Mbeya University of Science and Technology, P.O. Box 131, Mbeya, Tanzania.
[b] Department of Mathematics, Tanzania Institute of Accountancy, P.O.Box 9522, Dar es salaam, Tanzania.
[c] Department of Mathematics and Statistics, Sokoine University of Agriculture, P.O. BOX 3038, Morogoro, Tanzania.
[d] Department of General Studies, Dar es salaam Institute of Technology, P.O. Box 2958, Dar es Salaam, Tanzania.

• *Received: 23 June 2025* • *Accepted: 30 July 2025* • *Published Online: 05 November 2025*

Abstract

We introduce a Susceptible-Corrupted-Paying Well-Recovered (SCPwR) model specifically designed to analyze corruption dynamics within the public sector. This model is demonstrated to be well posed both epidemiologically and mathematically. Our results show that all model solutions remain positive and bounded given initial conditions within a meaningful set. We investigate the existence of unique corruption-free and endemic equilibrium points and calculate the basic reproduction number. The local and global stability of these equilibrium points is then analyzed. Our findings indicate that the system has a locally asymptotically stable corruption-free equilibrium when $R_e < 1$ and unstable when $R_e > 1$. Additionally, the corruption endemic equilibrium point (E^*) is globally asymptotically stable only if $\frac{d\mathcal{L}}{dt} < 0$. Numerical implementation of the model suggests that corruption will persist in public sectors if civil servants are not adequately compensated.

Keywords: Corruption, Dynamics, Public Sectors, Optimal Control.
2010 MSC: 34C60, 93A30.

1. Introduction

According to the Corruption Perception Index (CPI) [1], which incorporates indicators such as bribery, diversion of public funds, officials' misuse of public office for personal gain without facing repercussions, governments' effectiveness in combating corruption in the public sector, nepotistic practices within the civil service, state capture by narrow interests,

*Corresponding author: kigodimadeullah@gmail.com

bureaucratic hurdles in the public sector, legal protection for corruption whistle blowers, among others, corruption levels are measured on a scale of 0 to 100. A score of 0 denotes high corruption, while 100 represents a minimal level of corruption. The data reveals that Sub-Saharan Africa, with a score of 32, ranks as the most corrupt region globally, whereas Western Europe and the European Union, scoring 66, are perceived as relatively free from corruption.

Corruption is a pervasive and harmful phenomenon that undermines the integrity and effectiveness of public sectors worldwide [3]. It encompasses a range of unlawful activities carried out for personal gain, often through the abuse of authority or power by public officers or private individuals [4]. Corruption occurs when someone in charge, such as a government official or the company leaders, use their power unfairly to benefit themselves instead of fulfilling their responsibilities properly [5]. corruption is not just a consequence but a source of conflict, catalysing it in several ways [6]. Corruption poses a significant threat to any economy, particularly in low-income contexts [7]. Tanzi and Davoodi [8], in their study, observed that corruption leads to a rise in public investment levels at the detriment of private investments due to opportunities for manipulating public expenditure.

Corruption manifests in diverse ways, encompassing bribery, embezzlement, favoritism, nepotism, extortion, and other forms [9]. Its detrimental impact extends globally, posing significant challenges to the effective functioning of public sectors by eroding trust in institutions, distorting resource allocation, and hindering economic development [10]. Corruption undermines governance, fosters exemptions, and corrodes the integrity of governmental processes, perpetuating systemic inefficiencies [11]. Moreover, technological advancements have facilitated the emergence of new forms of corruption, such as cybercrime and digital bribery, adding complexity to efforts aimed at combating corrupt practices within the public sector [12]. Despite the widespread implementation of anti-corruption policies or strategies across many countries, corruption remains a persistent societal issue, indicating the challenges in effectively addressing its root causes [13]. The government needs to strengthen regulations to ensure proper adherence to due process and be ready to enforce laws against offenders [14]. Nevertheless, while these measures can be effective, devising robust intervention strategies to combat corruption and corrupt practices necessitates a comprehensive grasp of corrupt processes, prevention initiatives, and disengagement programs, which involve intricate mathematical modeling [15].

The role of mathematical modeling and optimal control analysis in understanding corruption transmission dynamics and guiding intervention program decisions is paramount [16]. Various mathematical modeling approaches have been proposed to investigate corruption dynamics, each providing distinct insights into the underlying mechanisms and transmission pathways of corrupt practices. For instance, Teklu's study [17] on insight into the optimal control strategies on corruption dynamics using fractional order derivatives revealed that the implementation of three time-dependent controlling strategies significantly reduces the number of corrupted individuals in the community. Conversely, a study by Keno and Legesse [37] on modeling and optimal control strategies of corruption dynamics determined that prevention emerges as the optimal and most cost-effective strategy for eradicating corruption. Additionally, Olaniyi et al.'s study [19] on optimal control and cost-effectiveness analysis of illicit drug use population dynamics highlighted that a single implementation of preventive control stands as the most cost-effective intervention for managing corruption dynamics. Hathroubi and Trabelsi [20] examined corruption as an epidemic phenomenon, employing the epidemic diffusion model originally proposed by Kermack and Mc-Kendrick [21].

Similarly, the study conducted by Alemneh [16] on mathematical modeling, analysis, and optimal control of corruption dynamics concluded that an integrated control strategy is essential to combat corruption effectively. Fantaye and Birhanu [22] developed a study on mathematical model and analysis of corruption dynamics with optimal control, revealing that the prevention and punishment strategy is the most effective approach to reducing the dynamic transmission of corruption. Nathan et al. [23] developed a novel mathematical model to depict the dynamics of moral corruption, incorporating age-appropriate sexual information and offering guidance and counseling. Danford [24] investigated corruption dynamics in Tanzania, focusing on modeling the effects of control strategies. The study recommended investing more in the provision of mass education to citizens, including general awareness campaigns and incorporating anti-corruption education into the curriculum from pre-primary to university levels. Additionally, leveraging religious leaders to educate their followers about the impacts of corruption was highlighted as a crucial measure. Numerical simulations show that effective interventions can reduce the spread of corruption in public institutions. Deterministic mathematical models are strongly recommended for studying endemic diseases [25, 26]. Integer and fractional-order models, as highlighted by [30, 31, 32], are essential tools in analyzing these diseases.

The study done by Shatanawi et al. [27] on approximate solutions for various fractional orders reveal that the susceptible and exposed classes decline more rapidly at lower fractional orders, slowing as they approach integer values. The infection rate increases, reducing the exposed class more quickly at integer orders and more slowly at fractional ones. This rise in infection also decreases the under-treatment class, with similar trends observed in the recovered class. Hussain et al. [28] formulated a stochastic epidemic model to evaluate disease persistence. Their results indicate that when noise levels are high, $R_0 < 1$, suggesting the disease does not persist. Conversely, the findings show that the disease persists only if the stochastic basic $R_0 > 1$. Abdo et al. [29] studied the spread of highly contagious diseases through direct contact and found that vaccination programs are the most effective means of preventing measles. They highlight that backward bifurcation can lead to inaccurate predictions if relying solely on the basic reproduction number (R_0), as measles can still occur even when $R_0 < 1$. Despite advancements in modeling, key issues for future research include the role of IgG antibodies in maternal immunity transfer and the significant impact of measles co-infection with other severe diseases like pneumonia and diarrhea.

1.1. Contribution of the Study

This study develops a comprehensive mathematical model to analyze the dynamics of corruption within public sectors and explores optimal control strategies for mitigating its impact. Distinct from previous models, this research incorporates detailed mechanisms of corruption propagation alongside various intervention strategies, presenting a novel approach to understanding and controlling corrupt practices. Additionally, this study aims to offer practical insights for policymakers, enhancing the effectiveness of anti-corruption measures. By providing a framework for evaluating and implementing optimal control strategies, this research supports broader efforts to improve governance and reduce the socio-economic costs associated with corruption.

2. MATERIAL AND METHODS

2.1. Formulation of the Model

We introduce a Susceptible-Corrupted-Paying Well-Recovered (SCPwR) model specifically tailored to examine the dynamics of corruption within public sectors, serving as the focal point of our investigation. The model is based on the following assumptions: Firstly, it employs a simplified mass action framework. Secondly, it operates under the assumption of enduring resistance against corruption. Thirdly, it incorporates a constant recruitment rate for employees. Fourthly, it assumes a transition rate from the susceptible class to the corrupted class denoted by α, and a transition rate from the susceptible class to the well-compensated class denoted by β. Additionally, it assumes a transition rate from the corrupted class to the well-compensated class denoted by δ. Moreover, the model considers a recovery rate resulting from being well-compensated, labeled as ϕ, alongside a natural recovery rate denoted by θ. Furthermore, it takes into account a natural mortality rate denoted by μ. These assumptions collectively underpin the dynamics of corruption and optimal control within public sectors as modeled. Table 1 indicates the descriptions of parameters and variables used in model 2.1.

Table 1: Description of variables and parameters used in Model 2.1

Variable/Parameter	Description
S	Susceptible individual
C	Corrupted individual
P_w	Well-compensated individual
R	Recovered individual
Λ	Recruitment rate
μ	Natural mortality rate
α	Rate of infection
θ	Natural recovery rate
β	Rate of transition from susceptible to well-compensated
δ	Rate of transition from corrupted to well-compensated
ϕ	Recovery rate due to being well-compensated

2.2. Model Flow diagram

Derived from the dynamics of corruption within public sectors, alongside model assumptions, variable definitions, and parameter specifications, the progression of corruption can be synthesized into Figure 1 as follows:

2.3. Model Equations

From Figure 1 we have the following linear differential equations:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \alpha SC - (\beta + \mu)S \\ \dfrac{dC}{dt} = \alpha SC - (\mu + \delta + \theta)C \\ \dfrac{dP_w}{dt} = \beta S + \delta C - (\mu + \phi)P_w \\ \dfrac{dR}{dt} = \theta C + \phi P_w - \mu R \end{cases} \tag{2.1}$$

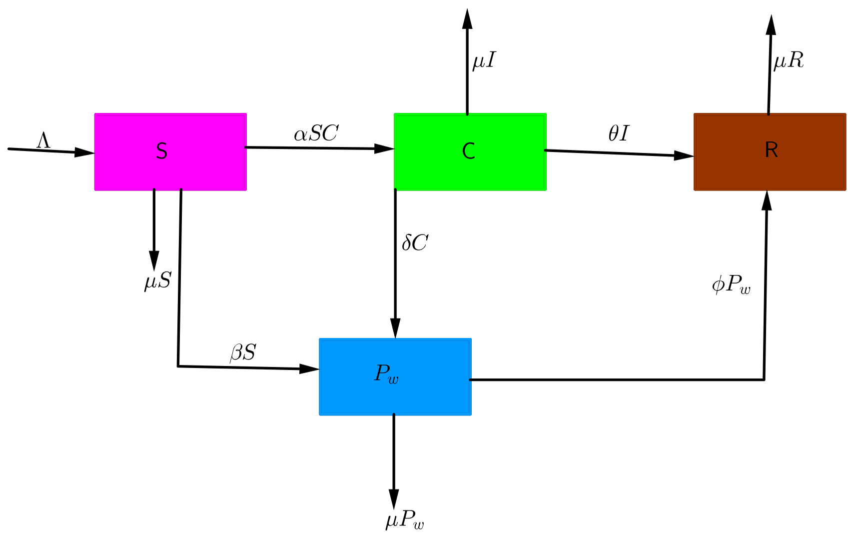


Figure 1: Compartment model diagram for the dynamics of Corruption and Optimal Control in Public Sectors.

with initial conditions;

$$S(0) > 0, \quad C(0) \geqslant 0, \quad P_w(0) \geqslant 0, \quad \textbf{and} \quad R(0) \geqslant 0. \tag{2.2}$$

2.4. Basic properties of the model

2.4.1. Invariant region

This region defines the state space in which the solutions of the differential equations representing the disease dynamics persist for all time $t \geqslant 0$ [2, 44, 45]. Suppose that the whole population is given by $N(t) = S(t) + C(t) + P_w(t) + R(t)$. It follows that:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dC}{dt} + \frac{dP_w}{dt} + \frac{dR}{dt}. \tag{2.3}$$

Substituting the equations of model system (2.1), into equation (2.3), we have

$$\begin{aligned} &\frac{dN}{dt} \leqslant \Lambda - \mu(S + C + P_w + R), \\ &\Longrightarrow \frac{dN}{dt} \leqslant \Lambda - \mu N, \\ &\Longrightarrow \frac{dN}{dt} + \mu N \leqslant \Lambda. \end{aligned} \tag{2.4}$$

Applying the approach of integrating factor and the initial condition given on (2.2) we get

$$N(t) \leqslant \frac{\Lambda}{\mu} + N(0)e^{-\mu t}. \tag{2.5}$$

Introducing limit as $t \to \infty$, it results

$$N(t) \leqslant \frac{\Lambda}{\mu}. \tag{2.6}$$

2.4.2. Positivity of Solution

Theorem 2.1 Let the initial conditions of the model variables in the equation (2.1) be $S(0) > 0, \quad C(0) \geqslant 0, \quad P_w(0) \geqslant 0,$ **and** $R(0) \geqslant 0$, then the solution set

$$\{S(0) > 0, \quad C(0) \geqslant 0, \quad P_w(0) \geqslant 0, \quad R(0) \geqslant 0\} \in \mathbb{R}^4_+ > 0$$

.

Proof. From the first equation of model (2.1),

$$\begin{aligned} \frac{dS}{dt} &= \Lambda - \alpha SC - (\beta + \mu)S, \\ \implies \frac{dS}{dt} &> -(\beta + \mu)S. \end{aligned} \tag{2.7}$$

Separation of variables and integration gives

$$\begin{aligned} \frac{dS}{S} &> -(\beta + \mu)dt, \\ \implies \int_{S(0)}^{S(t)} \frac{dS}{S} &> -\int_0^t (\beta + \mu)ds \\ \implies S(t) &> S(0)e^{-(\beta+\mu)} \geqslant 0. \end{aligned} \tag{2.8}$$

Applying the similar technique, it reveals that $C(t) \geqslant 0$, $P_w(t) \geqslant 0$, and $R(t) \geqslant 0$. □

2.5. Existence of the Corruption Free Equilibrium point (CFE)

When there is no corruption in the public institution, the corruption free equilibrium E^0, is given as: $E^0(S, C, P_w, R) = \left(\frac{\Lambda}{\mu + \beta}, 0, \frac{\Lambda\beta}{(\mu + \beta)(\mu + \phi)}, 0\right)$.

2.5.1. The effective reproduction number R_e

The basic reproduction number is the number of new infections that may arise when one infected individual is introduced in a population where there is intervention [41]. To obtain the effective reproduction number R_e, we adopt the approach of next generation matrix as used in [2, 43]. Let F be the force of infection and ν be the transfer terms, then from the model system 2.1, the values of F and V at disease free equilibrium are given as;

$$F = \frac{\alpha\Lambda}{\beta + \mu}$$

$$\nu = (\mu + \delta + \theta)C \implies V = \frac{d\nu}{dC} V = \mu + \delta + \theta \implies V^{-1} = \frac{1}{\mu + \delta + \theta}$$

Then,

$$\begin{aligned} FV^{-1} &= \frac{\alpha\Lambda}{\beta + \mu}\left(\frac{1}{\mu + \delta + \theta}\right) \\ \implies R_e &= \frac{\alpha\Lambda}{(\beta + \mu)(\mu + \delta + \theta)} \end{aligned}$$

2.5.2. Global stability of CFE

Let the Lyapunov function $\mathcal{L}$ be given as;

$$\begin{aligned}
\mathcal{L} &= \frac{1}{(\mu+\delta+\theta)}C \\
\implies \frac{d\mathcal{L}}{dt} &= \frac{1}{(\mu+\delta+\theta)}\frac{dC}{dt}, \\
\implies \frac{d\mathcal{L}}{dt} &= \frac{1}{\mu+\delta+\theta}\left(\alpha S-(\mu+\delta+\theta)\right)C.
\end{aligned}$$

But, at Corruption free equilibrium, $S^0 = \dfrac{\Lambda}{\beta+\mu}$. Thus we have:

$$\begin{aligned}
\frac{d\mathcal{L}}{dt} &= \frac{1}{\mu+\delta+\theta}\left(\frac{\alpha\Lambda}{\beta+\mu}-(\mu+\delta+\theta)\right)C, \\
&= \frac{\alpha\Lambda-(\beta+\mu)(\mu+\delta+\theta)}{(\beta+\mu)(\mu+\delta+\theta)}C, \\
\implies \frac{d\mathcal{L}}{dt} &= (R_e-1)C.
\end{aligned} \tag{2.9}$$

Thus, we conclude the following

Case I: The CFE is globally asymptotically stable if and only if $\frac{dV}{dt} < 0$ where $R_e < 1$ and $R_e = 0$.

Case II: The CFE is globally unstable if and only if $\frac{dV}{dt} > 0$ where $R_e > 1$.

Case III: The CFE is neither globally asymptotically stable nor unstable if and only if $\frac{dV}{dt} = 0$ where $R_e = 1$ and $C = 0$.

2.6. Existence of Corruption Endemic Equilibrium point

The corruption endemic equilibrium denotes a stage at which the corruption continues within the public sector. It is achieved by equating the model system's derivative to zero and then determining the state variables. Hence, the endemic equilibrium is represented as $E^\star(S^\star, C^\star, P_w^\star, R^\star)$ where:

$$\begin{aligned}
S^\star &= \frac{\mu+\delta+\theta}{\alpha}, \\
C^\star &= \frac{\theta\Lambda-(\beta+\mu)(\mu+\delta+\theta)}{\alpha(\mu+\delta+\theta)}, \\
P_w^\star &= \frac{\Lambda\alpha\delta+(\mu+\phi)(\delta+\theta+\mu)-\delta(\mu+\beta-1)(\delta+\theta+\mu)}{\alpha(\mu+\phi)(\delta+\theta+\mu)},
\end{aligned} \tag{2.10}$$

$$R^\star = \frac{\Lambda\alpha\delta(\delta\phi+\theta(\mu+\phi))-\phi\delta^2(\mu+\beta-1)(\delta+\theta+\mu)}{\alpha\mu\delta(\mu+\phi)(\delta+\theta+\mu)}$$
$$+\frac{\delta(\mu+\phi)(\delta+\theta+\mu)(\theta(\mu+2\beta-1)-\phi)+\theta(\mu+\phi)(\delta+\theta+\mu)(\beta(\theta+\mu)-(\phi+\mu))}{\alpha\mu\delta(\mu+\phi)(\delta+\theta+\mu)}.$$

2.7. Global Stability of Corruption Endemic Equilibrium Points

The global stability of the corruption endemic equilibrium point (E^*) was examined using the Lyapunov function. In accordance with the Lyapunov function a point is considered to be asymptotically globally stable if the derivative of the function is negative .

Theorem 2.1. *The corruption dynamics has a unique endemic equilibrium point E^* for the model system that is globally asymptotically stable if $R_e > 1$ and unstable otherwise.*

Proof. A Lyapunov function of the model system (2.1), as described by Vargas-De-León [33], Korobeinikov et al. [34], and Korobeinikov[35], was employed in this study. The Lyapunov function $\mathcal{L}$ is defined by

$$\mathcal{L}(\mathbf{y}) = \sum_{i=1}^{n} \frac{1}{2}(\mathbf{y}_i - \mathbf{y}_i^\star)^2,$$

Here, the population of the i-th compartment is denoted by $\mathbf{y}_i$, while $\mathbf{y}_i^\star$ designates the endemic equilibrium point.
The model system (2.1) exhibits the following positive definite function

$$P(S, C, P_w, R) = \sum_{i=1}^{4} \frac{1}{2}(\mathbf{y}_i - \mathbf{y}_i^\star)^2,$$

Then the above Lyapunov function of the corruption model system is written as:

$$\mathcal{L} = \frac{1}{2}[(S - S^\star) + (C - C^\star) + (P_w - P_w^\star) + (R - R^\star)]^2.$$

Then, differentiation of the function $L(t)$ with respect to time results in:

$$\frac{d\mathcal{L}}{dt} = [(S - S^\star) + (C - C^\star) + (P_w - P_w^\star) + (R - R^\star)]\frac{d}{dt}[S + C + P_w + R].$$
$$\frac{d\mathcal{L}}{dt} = [(S - S^\star) + (C - C^\star) + (P_w - P_w^\star) + (R - R^\star)]\frac{d}{dt}[S + C + P_w + R].$$

But,

$$\frac{d}{dt}(S + C + P_w + R) = \Lambda - \mu N.$$

And,

$$\Lambda - \mu N^\star = 0,$$
$$\Rightarrow \Lambda - \mu(S^\star + C^\star + P_w^\star + R^\star) = 0,$$
$$(S^\star + C^\star + P_w^\star + R^\star) = \frac{\Lambda}{\mu}.$$

Substituting into $\frac{d\mathcal{L}}{dt}$ gives

$$\frac{d\mathcal{L}}{dt} = [N(t) - \frac{(\Lambda)}{\mu}][\Lambda - \mu N(t)].$$

$$\frac{d\mathcal{L}}{dt} = [N(t) - \frac{(\Lambda)}{\mu}][-\mu(N(t) - (\Lambda))].$$
$$\frac{d\mathcal{L}}{dt} = -\mu[N(t) - \frac{\Lambda}{\mu}][N(t) - \frac{\Lambda}{\mu}].$$
$$\frac{d\mathcal{L}}{dt} \leqslant -\mu[N(t) - \frac{\Lambda}{\mu}]^2 < 0.$$

Thus, it is clear that $\frac{d\mathcal{L}}{dt} < 0$.
Therefore, the corruption endemic equilibrium point (E^*) is globally asymptotically stable. □

3. NUMERICAL SIMULATION OF THE MODEL

3.1. *Parameters values*

The table 2 indicates the name of the parameter used in the model, the value of the parameter, and the source from which the value has been extracted.

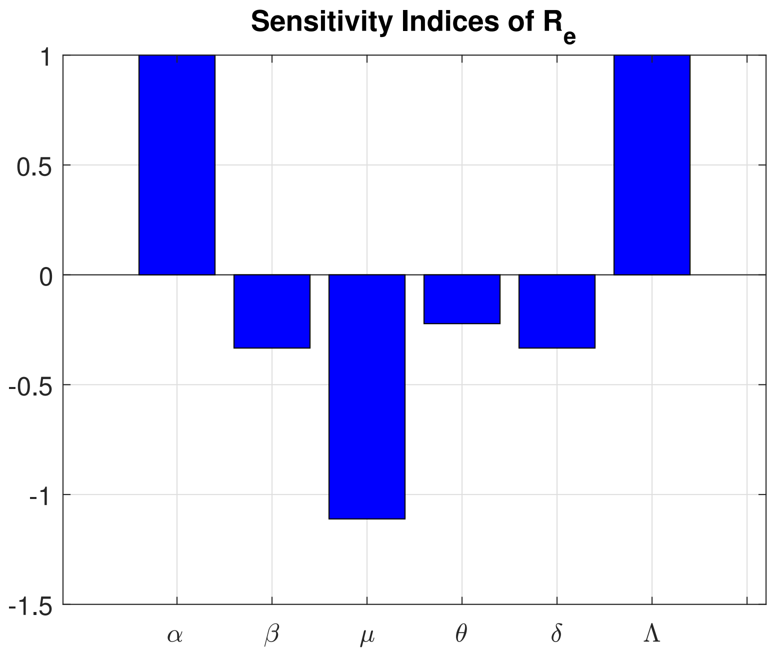
Table 2: Model parameters and their sources

Parameter	Value	Source
Λ	50	[36]
μ	0.02	[36]
α	0.024	[37]
β	0.01	Assumed
δ	0.015	Assumed
ϕ	0.010	[38]
θ	0.01	[39]

3.2. *Sensitivity analysis of* R_e

The sensitivity indices of the model parameters are calculated for determining parameters that have a major effect on effective reproduction numberR_e and should be targeted by strategies for intervention. The normalized forward sensitivity index method, as described by Chitnis *et al.*[40], is used to derive sensitivity indices. If $\mathcal{P}$ is a parameter in the effective reproduction number R_e, then its sensitivity index is given by:

$$\Gamma_{\mathcal{P}}^{R_e} = \frac{\partial R_e}{\partial \mathcal{P}} \times \frac{\mathcal{P}}{R_e}.$$



.

Figure 2: Sensitivity indices of R_e with respect to the parameters.

Referring to the graph presented in Figure (2), it illustrates the parameters affecting the value of R_e. Parameters with positive indices contribute to an increase in the value of R_e, while those with negative indices lead to a decrease in R_e when their values are raised. Parameters exhibiting the most positive sensitivity are the force of infections of corruption (α) and the recruitment rate (Λ), while the parameter showing the most negative sensitivity is the natural mortality rate (μ). Additionally, the rate at which corrupted individuals undergo natural recovery (θ) is depicted as the parameter with the smallest negative sensitivity. Consequently, it can be inferred that parameters α and Λ exert a positive influence on the effective reproduction number R_e, whereas μ exerts a negative influence on R_e. Furthermore, the index associated with parameter θ suggests that corruption poses a significant threat, with minimal probability for corrupted individuals to naturally recover. Based on these findings, it is recommended that implementing compensation mechanisms for infected and susceptible individuals should be considered as part of strategies aimed at controlling the transmission of corruption.

3.3. The effect of corruption interventions

To evaluate the impact of adequate compensation for public servants on reducing corruption within society, we conduct numerical simulations using the model system (2.1). Initially, we prioritize examining simulations for parameters with the most significant influence. Furthermore, we provide a discussion of the simulation results and their implications.

3.3.1. *The Impact of Varying Rate of Infection α on Susceptible Individuals*

Figure (3) illustrates that as the corruption rate rises, the number of susceptible individuals decreases because many of them transition to the corrupted class. Thus, a higher corruption rate leads to an increase in the number of corrupted individuals.

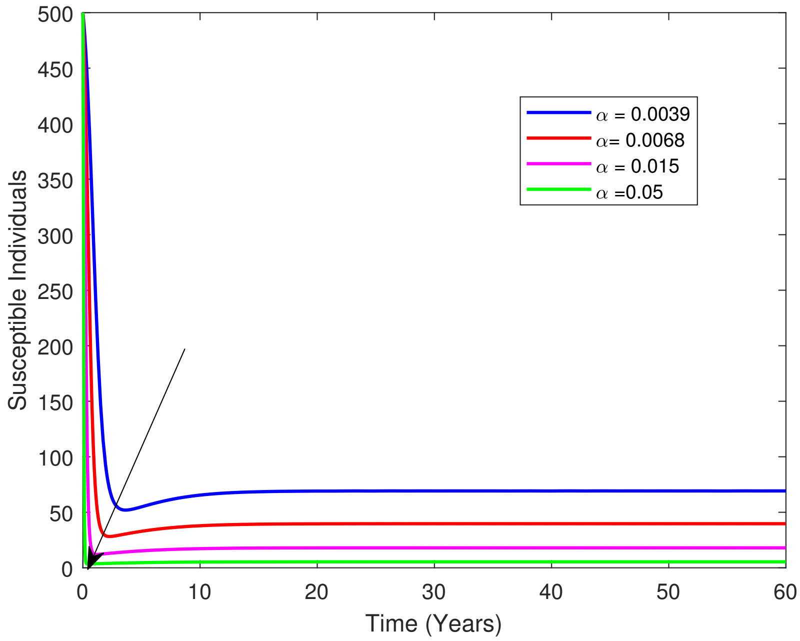


Figure 3: Variation of infection rate α on susceptible individuals

3.3.2. *The Impact of Varying δ on Corrupted Individuals*

Figure (4) demonstrates that with increase of the rate of payment δ, there is a decrease in the number of corrupted individuals as many of them move into the well-compensated class. Therefore, providing adequate payment to public servants significantly contributes to reducing corruption.

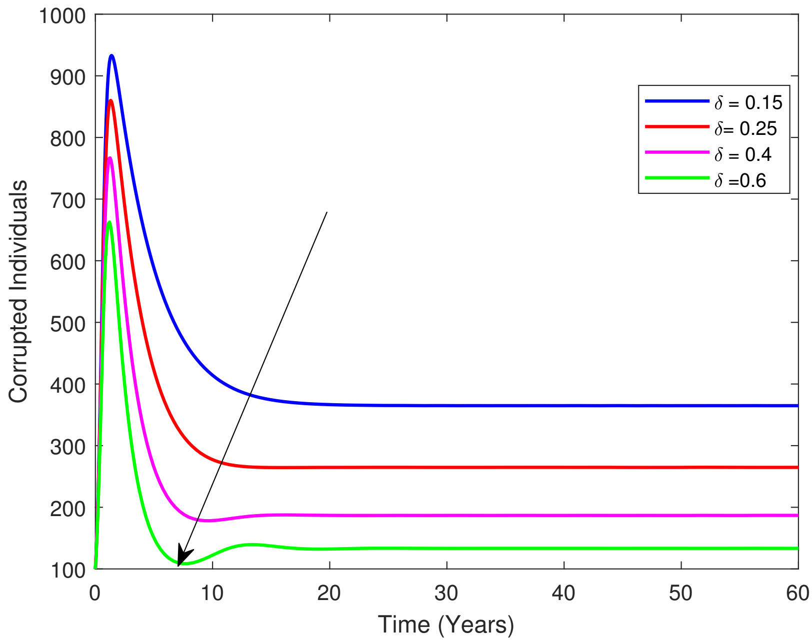


Figure 4: Variation compensation rate δ on corrupted individuals

3.3.3. The Impact of Varying β on Susceptible Individuals

Figure (5) depicts that as the well compensation rate β increases, there is a decrease in the number of susceptible public servants who are at risk of joining the corrupted individuals' class, as many of them transition to the well-compensated class. Therefore, ensuring adequate payment to public servants significantly contributes to reducing corruption.

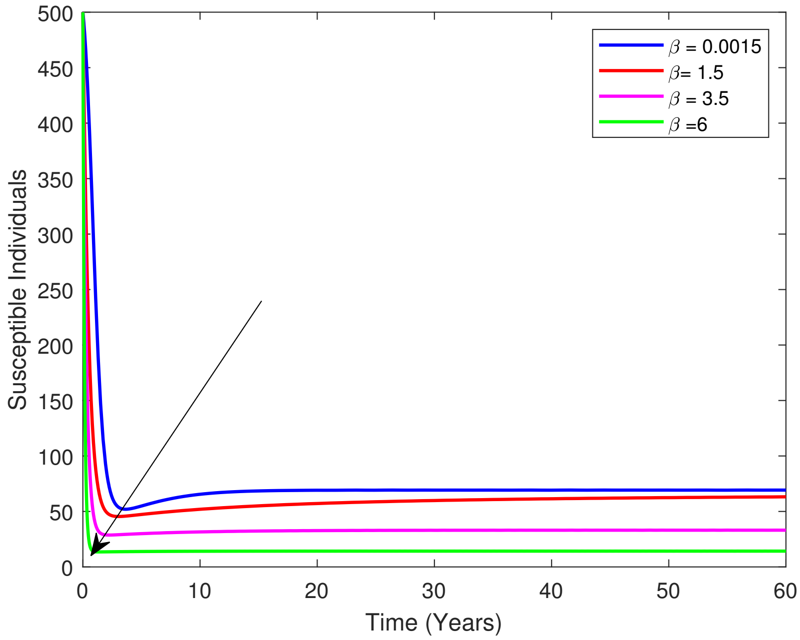


Figure 5: Variation of compensation rate β on susceptible individuals

3.3.4. Numerical Simulation of SCPwR Model

In this subsection, we perform numerical simulation of the model system (2.1). The initial conditions utilized to generate Figure (5) are as follows: $S(0) = 1000$, $C(0) = 100$, $Pw(0) = 5$, and $R(0) = 0$. These variables denote the initial quantities of susceptible, corrupted, paid well, and recovered individuals, respectively.

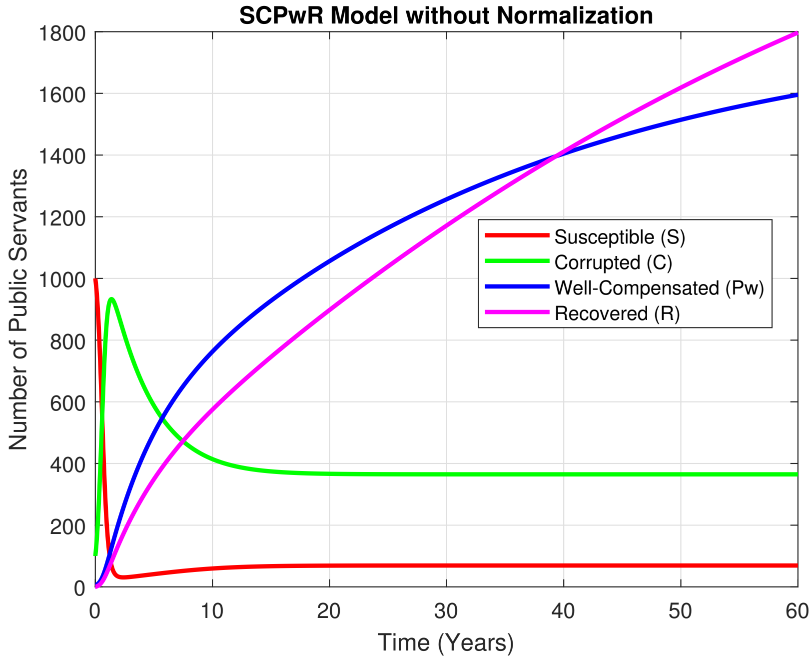


Figure 6: Dynamics of corruption drawn using the parameter values in Table 2 when there is an intervention

Figure (6) illustrates that, the susceptibility of public servants to corruption undergoes an initial sharp decline within the first 2 years due to the presence of high corruption rates across the entire population. However, this decline stabilizes over time, reaching a steady state. This trend, results to the fast increase for the number of corrupted individuals in the first 2 years. But, from the 2^{nd} year to the 10^{th} year, this number exponentially declines as many corrupted individuals transition to compensated class because of well payment. Additionally, the curves representing well-paid and recovered individuals exhibit continuous growth over time. The curve for well-paid individuals sees an increase through the recruitment of susceptible individuals and those transitioning from the corrupted class. Furthermore, after 40 years, the simulation indicates that the number of recovered individuals exceeds that of well-paid individuals. It is attributed by the grounds that corrupted individuals receiving better compensation are less likely to return to corrupt practices, leading to a sustained increase in the recovered population.

4. CONCLUSION AND RECOMMENDATIONS

4.1. Conclusion

The simulation highlights an initial decline and subsequent stabilization in the susceptibility of public servants to corruption over time. It underscores the importance of addressing corruption early on to prevent its proliferation. Additionally, the transition of corrupted individuals to well-paid positions plays a significant role in reducing corruption levels. Moreover, sustained efforts in providing better compensation lead to a higher number of recovered, proving that encouraging honesty is effective.

4.2. Recommendations

Implementing anti-corruption measures early on to capitalize on the initial decline in susceptibility, while prioritizing the transition of corrupted individuals to well-paid positions to mitigate corruption levels. Sustain efforts in providing competitive compensation to deter individuals from returning to corrupt practices, and regularly monitor and evaluate the effectiveness of anti-corruption policies to adapt strategies accordingly.

Acknowledgment

We express sincere gratitude to everyone who played a significant role in completing this project. Special thanks to our advisors for their invaluable guidance in shaping our simulations and sensitivity analyses. We appreciate those who participated in discussions, provided feedback, and shared their expertise, which greatly enriched the paper. This collaborative endeavor stands as a testament to their support.

References

[1] Transparency International (2022). Corruption. *The global coalition against corruption*. https://www.transparency.org/cpi
[2] Adeniran, A. O., Olanegan, O. O., & Akinsola, O. S. (2022). Mathematical Modeling: A Study of Corruption among Students of Nigeria Tertiary Institutions. *Journal of Mathematical Analysis and Modeling*, 3(1), 39-49.
[3] de Dieu Basabose, J. (2019). *Anti-corruption education and peacebuilding: The Ubupfura project in Rwanda*. Springer.
[4] Vargas-Hernández, J. G. (2009). The multiple faces of corruption: Typology, forms and levels. *Organizational immunity to corruption: Building theoretical and research foundations*, 111-117.
[5] Mungiu, A. (2006). Corruption: Diagnosis and treatment. *Journal of democracy*, 17(3), 86-99.
[6] Deglow, A., & Fjelde, H. (2021). The quality of government and civil conflict.
[7] Nwajeri, U. K., Asamoah, J. K. K., Ugochukwu, N. R., Omame, A., & Jin, Z. (2023). A mathematical model of corruption dynamics endowed with fractal–fractional derivative. *Results in Physics*, 52, 106894.
[8] Tanzi, V., & Davoodi, H. (2012, December). Corruption, Public In-vestment, and Growth. In *The Welfare State, Public Investment, and Growth: Selected Papers from the 53rd Congress of the International Institute of Public Finance* (p. 41). Springer Science & Business Media.
[9] Morris, S. D. (2011). Forms of corruption. *CESifo DICE report*, 9(2), 10-14.
[10] Julius Otusanya O (2011) Corruption as an obstacle to development in developing countries: a review of literature. *Journal of Money Laundering Control*. 14(4):387-422. https://doi.org/10.1108/13685201111173857
[11] Costa M (2023) Political Corruption: Causes, Consequences, and Mitigation Strategies. *Journal of Philosophical Criticism*. 6(02):136-150.
[12] Shelley LI (1998) Crime and corruption in the digital age. *Journal of International Affairs*. Pages 605-620. JSTOR.
[13] Persson A, Rothstein B, Teorell J (2010) The failure of anti-corruption policies: a theoretical mischaracterization of the problem.
[14] Nwabuzor A (2005) Corruption and development: new initiatives in economic openness and strengthened rule of law. *Journal of Business Ethics*. 59:121-138.
[15] Rotberg RI (2019) *The corruption cure: How citizens and leaders can combat graft*. Princeton University Press.
[16] Alemneh HT (2020) Mathematical modeling, analysis, and optimal control of corruption dynamics. *Journal of Applied Mathematics*. 2020:1-13. Hindawi Limited.
[17] Teklu SW (2024) Insight into the optimal control strategies on corruption dynamics using fractional order derivatives. *Scientific African*. 23:e02069. Elsevier.

[18] Keno TD, Legesse FM (2023) Modelling and optimal control strategies of corruption dynamics. *Applied Mathematics*. 17(4):673-683.

[19] Olaniyi S, Akanni JO, Adepoju OA (2023) Optimal control and cost-effectiveness analysis of an illicit drug use population dynamics. *Journal of Applied Nonlinear Dynamics*. 12(01):133-146. L&H Scientific Publishing.

[20] Hathroubi S, Trabelsi H (2014) Epidemic corruption: a bio-economic homology.

[21] Kermack WO, McKendrick AG (1927) A contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 115(772):700-721.

[22] Fantaye AK, Birhanu ZK (2022) Mathematical model and analysis of corruption dynamics with optimal control. *Journal of Applied Mathematics*. 2022:1-16. Hindawi Limited.

[23] Mokaya NO, Alemneh HT, Ngari CG, Muthuri GG (2021) Mathematical modelling and analysis of corruption of morals amongst adolescents with control measures in Kenya. *Discrete Dynamics in Nature and Society*. 2021:1-16. Hindawi Limited.

[24] Danford O (2021) Corruption dynamics in Tanzania: Modeling the effects of control strategies. NM-AIST.

[25] Mwasunda, J. A., Irunde, J. I., Kajunguri, D., & Kuznetsov, D. (2022). Optimal control analysis of Taenia saginata bovine cysticercosis and human taeniasis. *Parasite Epidemiology and Control*, 16, e00236.

[26] Mwasunda, J. A., Chacha, C. S., Stephano, M. A., & Irunde, J. I. (2022). Modelling cystic echinococcosis and bovine cysticercosis co-infections with optimal control. *Computational and Applied Mathematics*, 41(8), 342.

[27] Shatanawi, W., Abdo, M. S., Abdulwasaa, M. A., Shah, K., Panchal, S. K., Kawale, S. V., & Ghadle, K. P. (2021). A fractional dynamics of tuberculosis (TB) model in the frame of generalized Atangana–Baleanu derivative. *Results in Physics*, 29, 104739.

[28] Hussain, S., Madi, E. N., Khan, H., & Abdo, M. S. (2022) A numerical and analytical study of a stochastic epidemic SIR model in the light of white noise. *Advances in Mathematical Physics*.

[29] Abdo, M. S., Kamarany, M. A. A., Suhail, K. A., & Majam, A. S. (2022). Vaccination-based measles outbreak model with fractional dynamics. *Abhath Journal of Basic and Applied Sciences*, 1(2), 6-10.

[30] Shah, K., & Abdeljawad, T. (2023). On complex fractal-fractional order mathematical modeling of CO 2 emanations from energy sector. *Physica Scripta*, 99(1), 015226.

[31] Yadav, P., Jahan, S., Shah, K., Peter, O. J., & Abdeljawad, T. (2023). Fractional-order modelling and analysis of diabetes mellitus: Utilizing the Atangana-Baleanu Caputo (ABC) operator. *Alexandria Engineering Journal*, 81, 200-209.

[32] Ouncharoen, R., Shah, K., Ud Din, R., Abdeljawad, T., Ahmadian, A., Salahshour, S., & Sitthiwirattham, T. (2023). Study of integer and fractional order COVID-19 mathematical model. *Fractals*, 31(04), 2340046.

[33] Vargas-De-León C (2009) Constructions of Lyapunov functions for classic SIS, SIR and SIRS epidemic models with variable population size. *Foro-Red-Mat: Revista electrónica de contenido matemático*. 26(5):1-12.

[34] Korobeinikov A, Wake GC (2002) Lyapunov functions and global stability for SIR, SIRS, and SIS epidemiological models. *Applied Mathematics Letters*. 15(8):955-960. Elsevier.

[35] Korobeinikov A (2004) Lyapunov functions and global properties for SEIR and SEIS epidemic models. *Mathematical Medicine and Biology: A Journal of the IMA*. 21(2):75-83. OUP.

[36] Athithan, S., Ghosh, M., & Li, X.-Z. (2018). Mathematical modeling and optimal control of corruption dynamics. *Asian-European Journal of Mathematics, 11*(06), 1850090. World Scientific.

[37] Keno, T. D., & Legesse, F. M. (2023). Modelling and optimal control strategies of corruption dynamics. *Applied Mathematics, 17*(4), 673–683.

[38] Lemecha, L. (2018). Modelling corruption dynamics and its analysis. *Ethiopian Journal of Science and Sustainable Development, 5*(2), 13–27.

[39] Binuyo, A. O. (2019). Eigenvalue elasticity and sensitivity analyses of the transmission dynamic model of corruption. *Journal of the Nigerian Society of Physical Sciences*, 30–34.

[40] Chitnis, N., Hyman, J. M., & Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. *Bulletin of mathematical biology*, 70, 1272-1296.

[41] Diekmann, O., Heesterbeek, J. A. P., & Metz, J. A. J. (1990). On the definition and the computation of the basic reproduction ratio R0 in models for infectious diseases in heterogeneous populations. *Journal of Mathematical Biology*, 28, 365–382. Springer.

[42] Castillo-Chavez, C., Blower, S., van den Driessche, P., Kirschner, D., & Yakubu, A.-A. (2002). *Mathematical Approaches for Emerging and Reemerging Infectious Diseases: Models, Methods, and Theory* (Vol. 126). Springer Science & Business Media.

[43] Nyerere, N., & Liana, Y. A. (2024). A mathematical analysis of the effects of control strategies on the transmission dynamics of Chlamydiosis. *Decision Analytics Journal*, 100490. Elsevier.

[44] Guan, R., Kopfová, J., & Rachinskii, D. (2024). Global stability of SIR model with heterogeneous transmission rate modeled by the Preisach operator. *SIAM Journal on Applied Dynamical Systems*, 23(2), 1199-1241.

[45] He, S., Tan, Y., & Wang, W. (2024). Stochastic dynamics of an SIR model for respiratory diseases coupled with air pollutant concentration changes. *Advances in Continuous and Discrete Models*, 2024(1), 16.