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Modeling the Transmission Dynamics of Bushfires on Cashew Nut Production

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Abstract

Bushfires pose a significant threat to cashew nut production, impacting both agricultural yields and economic stability. This study developed a mathematical model to characterize the transmission dynamics of bushfires and their effects on cashew nut production. The basic bushfire transmission metric, $\mathbb{R}_0$, was computed and it was used to analyze the existence and stability of the equilibrium. Sensitivity analysis was conducted using the normalized forward sensitivity index method, identifying that an effective bushfire propagation rate significantly increases the incidence of bushfires. The findings indicated that reducing bushfire propagation and enhancing fire management practices can significantly mitigate the spread of bushfires, leading to substantial reductions in bushfire incidence. Consequently, effective control strategies targeting bushfire propagation rates and land management practices are crucial for safeguarding cashew nut production. This study contributes to a comprehensive understanding of bushfire dynamics and provides insights for developing targeted interventions to mitigate bushfire risks in cashew nut production. Future research directions may involve integrating additional environmental factors, weather patterns, and exploring scenario analysis to further refine bushfire management strategies.

Keywords: Modeling, Transmission, Bush fires, Dynamics, Cashew Nut.

1. Introduction

Bushfires, also known as wildfires, are exceptionally powerful and fast-moving fires that spread rapidly across vegetation such as forests, grasslands, or shrublands [1, 2]. These fires are characterized by intense heat, rapid spread with towering flames, and the ability to consume vast areas of land in a short period. They can be ignited by various factors including lightning strikes, human activities, or natural conditions like dry weather and high temperatures [3, 7].

The ecological impact of bushfires is profound. They disrupt nutrient cycling, destroy habitats, and alter plant community compositions, leading to long-term changes in biodiversity [35, 30]. Wildlife species dependent on specific vegetation types are particularly

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vulnerable, facing habitat loss and reduced food sources [30]. In human communities, bushfires pose health risks due to air pollution from smoke inhalation and ash particles, exacerbating respiratory conditions [9]. The increase in bushfire frequency and intensity in recent years is largely attributed to climate change, which contributes to hotter and drier conditions conducive to fire spread [32, 24]. Moreover, human activities such as land clearing and inadequate fire management practices can exacerbate fire risks, further threatening ecosystems and human settlements [24, 14].

Bushfires globally pose a significant threat to agricultural systems, particularly in Tanzania, where they severely impact cashew nut production, a cornerstone of the economy [8]. These fires harm cashew nut trees by causing physiological stress, reducing growth rates, and affecting reproductive processes, thereby compromising overall yield and quality [11]. Given the unique ecological and economic importance of cashew nut trees, understanding the biological consequences of bushfires is crucial for developing effective mitigation strategies [10]. Despite their critical role in supporting Tanzania's economy and farmers' livelihoods, the susceptibility of cashew nut trees to fire damage raises concerns about their resilience in the face of escalating bushfire frequency and severity.

Bushfires in Tanzania inflict significant damage on cashew nut trees, diminishing their productivity and health, and contributing to soil degradation that adversely affects tree growth and nut quality [1]. These fires disrupt natural growth cycles, impeding pollination and growth stages, resulting in irregular yields [12]. The shift in farmers' focus towards fire prevention disrupts critical farming practices, further jeopardizing tree health and ultimately reducing nut yield [26]. The economic losses suffered by farmers and the broader ecological disturbances underscore the extensive impacts of these fires, threatening both livelihoods and ecosystem sustainability in cashew-growing regions.

Effective bushfire control involves implementing firebreaks to create barriers that halt the spread of fires, managing fuel loads to reduce combustible materials, and employing early detection systems to promptly identify ignition sources [18]. Trained firefighting teams play a crucial role in suppressing fires before they escalate and cause extensive damage to ecosystems. Aerial support, such as water bombing and reconnaissance flights, aids in reaching and extinguishing remote fires that are inaccessible by ground teams [13]. Biologically, wildfires impact flora and fauna by altering habitats, affecting species diversity, and disrupting ecological processes. Community engagement and education are essential for promoting fire awareness, fostering a proactive approach to fire prevention, and ensuring swift responses in fire emergencies [18]. Additionally, monitoring fire weather conditions, enforcing regulations on fire safety practices, fostering collaborations among agencies, and advancing technology for fire prediction and management are indispensable strategies to mitigate the devastating effects of wildfires on biodiversity and ecosystem resilience [13].

Various mathematical models have been used to explore different real-life problems facing society [41, 40, 39]. In particular, these models serve as crucial tools in mitigating the impact of bushfires on cashew nut production by providing insights into fire behavior and its effects on orchards [4]. These models simulate fire spread patterns, predict potential damage to cashew nut trees, and assess the vulnerability of different regions to fires [5]. By integrating factors like fire frequency, weather conditions, soil properties, and tree physiology, these models enable the development of targeted fire prevention strategies, such as creating effective firebreaks or implementing controlled burns [6]. Researchers have turned to mathematical models to understand how bushfires impact cashew nut production in Tanzania.

As an illustration, Khastagir et al. [15] conducted an assessment of various techniques for parameter estimation in extreme bushfire modeling within Victoria, Australia. Millimono et al. [20] explored the modeling of bushfire spread in the Malea region of Northeastern Guinea, offering valuable insights into fire dynamics locally. Nguyen et al. [21] investigated the modeling of hazardous reduction burnings and bushfire emissions within an air quality framework, assessing their impact on health in the greater metropolitan area of Sydney. Rahman et al. [25] examined forest fire occurrence and modeling in Southeastern Australia. Additionally, Tayari et al. [27] proposed a conceptual framework for strategically selecting bushfire mitigation approaches. However, none of these studies addressed the impact of bushfires on cashew nut production, a gap that our research aims to fill.

2. Model Formulation

The overall population of cashew trees, denoted as $N(t)$, is categorized into three compartments: Susceptible ($S(t)$), representing healthy cashew nut trees capable of producing nuts and unaffected by bushfires; Infected ($I(t)$), signifying trees that have experienced fire-related injuries or stress, impacting their growth, vitality, and productivity; and Recovered ($R(t)$), representing trees that have recuperated from fire damage and are on their way to returning to a healthy state. Thus, the total population $N(t)$ is given by $N(t) = S(t) + I(t) + R(t)$.

Cashew trees are recruited into the population at a rate of Λ. This recruitment represents the natural germination and growth of new cashew trees into the population. When a bushfire event occurs, it causes trees in the Susceptible ($S(t)$) compartment to move to the Infected ($I(t)$) compartment at a rate of β, indicating the trees affected by the fire. These affected trees experience physiological stress and damage, including scorched leaves, bark damage, and reduced photosynthetic capacity, which impact their overall health and productivity.

Some trees in the Infected compartment recover from the fire damage and transition to the Recovered ($R(t)$) compartment at a rate of α. This recovery process involves biological repair mechanisms such as tissue regeneration, re-sprouting, and healing of fire wounds, indicating their successful recuperation and return to a healthy state capable of producing nuts again. After recovery, these trees might regain their susceptibility to fire damage at a rate of ϕ, representing the natural cycle of susceptibility and resistance in the tree population.

The overall tree population decreases due to natural deaths, represented by μ, which include senescence and disease, and fire-induced deaths, represented by δ, which account for the trees that succumb to severe fire damage and cannot recover. All these parameter explanations are detailed in Table 1.

2.1. Compartmental Model of Bushfire Spread

The interactions between susceptible, infected and recovered cashew trees are illustrated in Figure 1. Based on the model's assumptions and the flow chart in Figure 1, we

Table 1: Parameters of the model and their explanations

Parameter	Explanation	Value	Source
Λ	Cashew plants replanting rate	0.97	[22]
β	Effective bush-fire propagation rate	0.2	Assumed
μ	Cashew plants natural death rate	0.04	[22]
α	Recovery rate of Cashew plants	0.1	Assumed
ϕ	The rate of returning to susceptible state	0.5	[23]
δ	Fire induced death rate of Cashew plants	0.008	[29]

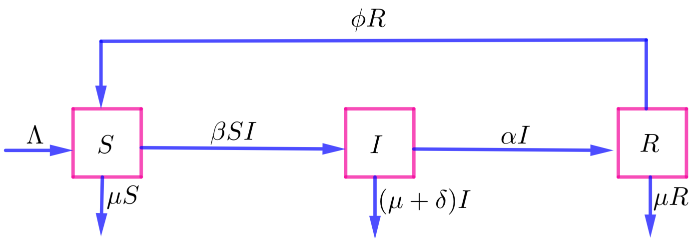

Figure 1: A schematic diagram illustrating the transmission dynamics of bush fires.

can formulate the following nonlinear ordinary differential equations:

$$\begin{aligned} \frac{dS}{dt} &= \Lambda + \phi R - (\beta I + \mu)S \\ \frac{dI}{dt} &= \beta SI - (\alpha + \delta + \mu)I \\ \frac{dR}{dt} &= \alpha I - (\phi + \mu)R \end{aligned} \tag{2.1}$$

where: $S(0) > 0$, $I(0) \geqslant 0$, and $R(0) \geqslant 0$.

3. Model properties

3.1. Positivity of solution

To ensure that all variables and outputs of the model remain non-negative throughout the system, we demonstrate the positivity of the solution. We use the approach which was also used by Abdo et al. [36] and Sher et al. [38].

Theorem 3.1. *If we define ψ as the set of initial conditions where $S(0)$ is strictly positive, and both $I(0)$ and $R(0)$ are non-negative, then the solution $(S(t), I(t), R(t))$ for the system described in (2.1) remains non-negative for all $t \geqslant 0$.*

Proof. From the first equation of model system (2.1), we have;

$$\begin{aligned}\frac{dS}{dt} &= \Lambda + \phi R - \mu S - \beta SI \\ \frac{dS}{dt} &\geqslant -\mu S \\ \frac{dS}{S} &\geqslant -\mu dt\end{aligned} \tag{3.1}$$

By separating the variables, we have;

$$\int_{s(0)}^{s(t)} \frac{dS}{S} \geqslant -\int_0^t \mu dt$$
$$\ln S(t) - \ln S(0) \geqslant -\mu t$$

Upon further simplification, we obtain:

$$S(t) > S(0)e^{-\mu t}$$

Moreover, employing similar procedures, we derive:

$$I(t) \geqslant I(0)e^{-(\alpha+\delta+\mu)t}$$
$$R(t) \geqslant R(0)e^{-(\phi+\mu)t}$$

Therefore, all solution sets exhibit positive values for $t \geqslant 0$. Therefore, the model is meaningful and well-posed. This well-posed characteristic of the model allows us to conduct additional mathematical analysis confidently as described by Abdo et al. [33, 40].

□

3.2. Stability region for bush fire transmission dynamics

In this subsection, we examine whether the model variables possess biological significance and ensure the existence of a unique, bounded solution over time as used in the study conducted by Abdo et al. [34]. From the model system (2.1), we derive:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = \Lambda - \mu N - \delta I. \tag{3.2}$$
$$\frac{dN}{dt} \leqslant \Lambda - \mu N$$

Upon solving this equation, we find:

$$0 \leqslant N(t) \leqslant \frac{\Lambda}{\mu} + N(0)e^{-\mu t}.$$

As $t \to \infty$, it becomes evident that $0 < N(t) \leqslant \frac{\Lambda}{\mu}$. Consequently, the model solution maintains its feasibility and positive invariance within the region:

$$\Omega = \left\{ (S, I, R) \geqslant 0 \in \mathbb{R}^3_+ : S + I + R \leqslant \frac{\Lambda}{\mu} \right\}.$$

The existence of a viable solution for the model, maintaining positivity within $\mathbb{R}^3_+$, signifies the model's soundness both from an epidemiological and mathematical standpoint. This robustness enables us to proceed with subsequent mathematical analyses with confidence.

3.3. Bush Fires-Free Equilbrium Point(BFE)

The Bush Fires-Free Equilibrium (BFE) point refers to a state in a model where no bushfires are present, indicating stability. It represents a scenario where the spread of bushfires ceases. We arrive at this equilibrium point by setting the model system (2.1) to zero and considering $I = 0$. Thus, the equilibrium point, denoted as E^0, representing a state without bush fires, is expressed as:

$$E^0 = (S^0, I^0, R^0) = \left(\frac{\Lambda}{\mu}, 0, 0\right)$$

3.4. Fundamental Fire Transmission Metric ($\mathbb{R}_0$)

The fundamental fire transmission metric ($\mathbb{R}_0$) signifies the average number of new fires generated by a single existing fire in an environment susceptible to fire, indicating its potential for transmission. This metric is pivotal in evaluating the extent and severity of bushfires, assisting in the formulation of fire management strategies and risk evaluation. A $\mathbb{R}_0$ value exceeding 1 indicates an increasing trend in fire spread, whereas values below 1 suggest a declining propagation. Determining the fundamental fire transmission metric $\mathbb{R}_0$ of the system involves applying the formula $\mathbb{R}_0 = (\text{spreading rate}) \times (\text{duration of fire spread})$, as outlined by Van den Driessche [17]. Initially, the model equations are rewritten, commencing with the newly infected classes:

$$\frac{dI}{dt} = \beta S I - (\alpha + \delta + \mu) I$$

Subsequently,

$$\mathbb{R}_0 = \beta \times \left(\frac{\Lambda}{\mu(\alpha + \delta + \mu)}\right) \tag{3.3}$$

Hence,

$$\mathbb{R}_0 = \frac{\beta \Lambda}{\mu(\alpha + \delta + \mu)} \tag{3.4}$$

3.5. The Stability of BFE at a Localized Level

In this subsection, the eigenvalue method is utilized to investigate the local stability of the bush fires-free equilibrium point in the model system (2.1).

Theorem 3.2. *The bush fires-free equilibrium for the model system (2.1) is locally asymptotically stable if $\mathbb{R}_0 < 1$ and unstable if $\mathbb{R}_0 > 1$.*

Proof. To demonstrate the local stability of the bush fires-free equilibrium, we aim to establish that the variational matrix $J(E_0)$ of the model system has only negative eigenvalues. The Jacobian matrix for the model system (2.1) is defined as:

$$J(E_0) = \begin{bmatrix} -\mu & -\beta\frac{\Lambda}{\mu} & \phi \\ 0 & \beta\frac{\Lambda}{\mu} - (\alpha + \delta + \mu) & 0 \\ 0 & \alpha & -(\phi + \mu) \end{bmatrix}$$

Here, $\lambda_1 = -\mu, \lambda_2 = \beta\frac{\Lambda}{\mu} - (\alpha + \delta + \mu), \lambda_3 = -(\phi + \mu)$. It is observed that, the Jacobian matrix $J(E_0)$ possesses three distinct eigenvalues: $\lambda_1 = -\mu, \lambda_2 = -(\alpha + \delta + \mu)(\mathbb{R}_0 - 1), \lambda_3 = -(\phi + \mu)$. All eigenvalues are negative if $\mathbb{R}_0 < 1$. Therefore, the bush fires-free equilibrium is locally asymptotically stable if $\mathbb{R}_0 < 1$.

3.6. Global Stability of Bush Fires-Free Equilibrium Point

In this part, we investigate the worldwide stability of the bushfire-free equilibrium point (BFE) point using the methodology outlined in the study conducted by Nyerere et al. [31]. The model system (2.1) can be represented in a streamlined manner as:

$$\frac{dX_s}{dt} = A(X_s - X_{DFE,S}) + A_1 X_i$$
$$\frac{dX}{dt} = A_2 X_i$$

where X_s denotes the vector for the non-fire-transmitting groups, and X_i denotes the vector for the fire-transmitting groups. The Bush Fires-Free Equilibrium Point (BFE) is globally asymptotically stable if matrix A has real negative eigenvalues and A_2 is a Metzler matrix (i.e., all off-diagonal elements of A_2 are non-negative). It must be verified that, the matrix A associated with the non-fire-transmitting groups has real negative eigenvalues and that A_2 is a Metzler matrix. From the model system (2.1), it can be identified that, $X_s = (S, R)^T$ and $X_i = (I)^T$. For the non-fire-transmitting groups, we have:

$$A = \begin{bmatrix} -\mu & \phi \\ 0 & -(\phi + \mu) \end{bmatrix}$$

with eigenvalues $\lambda_1 = -\mu$ and $\lambda_2 = -(\phi + \mu)$, and:

$$A_2 = \begin{bmatrix} \beta S^0 & -(\alpha + \delta + \mu) \end{bmatrix}$$

Observing that A_2 is a Metzler matrix and A has real negative eigenvalues, It can be concluded that, the bush fires-free equilibrium of the model system (2.1) is globally asymptotically stable. □

3.7. Steady State of the Model Supporting Bushfires

This refers to a condition within the model where the variables stabilize over time, maintaining the presence and propagation of bushfires without significant fluctuations. This state reflects the equilibrium point where the factors contributing to the occurrence and spread of bushfires reach a balance, resulting in a sustained presence of bushfires within the system. It can be obtained as follows:

$$\begin{aligned} \Lambda + \phi R^* - (\beta I + \mu) S^* &= 0 \\ \beta S^* I^* - (\alpha + \delta + \mu) I^* &= 0 \\ \alpha I^* - (\phi + \mu) R^* &= 0 \end{aligned} \tag{3.5}$$

From equation 3.5, we get

$$\begin{aligned} S^* &= \frac{1}{\mathbb{R}_0} \\ I^* &= \frac{\beta \mathbb{R}_0 (\phi + \mu)}{\phi\alpha - \mu(\phi + \mu) + \Lambda(\beta(\phi + \mu))} \\ R^* &= \frac{1}{\mathbb{R}_0} \times \frac{\alpha}{\phi + \mu} \end{aligned} \tag{3.6}$$

Thus, the steady state of the model exists if $\mathbb{R}_0 > 1$.

4. Findings and Discussion

4.1. Sensitivity Analysis

Understanding the relative significance of various factors contributing to the occurrence and spread of bush fires is crucial for mitigating the decline in cashew nut production. In the sensitivity analysis, we examine the impact of model parameters on both fire transmission and the basic reproduction number $\mathbb{R}_0$. Employing the normalized forward sensitivity index method, we derive an analytical expression to quantify the sensitivity of $\mathbb{R}_0$ to five parameters:

$$\gamma_{\beta}^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0} \tag{4.1}$$

Here, $\gamma_{\beta}^{R_0}$ denotes the sensitivity index, and β represents the parameter influencing the bush fire propagation number. Higher values of $\gamma_{\beta}^{R_0}$ indicate a greater impact on the bush fire propagation number. Consequently, the sensitivity indices for the parameters are outlined in Table 2.

4.2. Sensitivity Indices

The sensitivity indices quantify the impact of variations in each parameter on the basic bush fire propagation number $\mathbb{R}_0$. A higher sensitivity index indicates that small changes in the parameter value lead to larger changes in $\mathbb{R}_0$, highlighting the parameter's significance in driving the epidemic dynamics.

Table 2: Sensitivity Indices

Parameter	**Index**
Λ	+1.0000
α	+0.6757
β	+1.0000
δ	+0.0541
μ	–0.7297

4.3. Interpretation of the sensitivity indices

Positive sensitivity indices indicate parameters where an increase leads to a proportional increase in model outcomes. For example, a positive sensitivity index for parameters like the replanting rate of cashew nut trees (Λ) and the effective contact rate (β) suggests that higher values of these parameters result in an amplified spread of bushfires and damage to cashew nut production. For example, an increase in the bush fire propagation rate β leads to a direct and proportional rise in the spread of bushfires and the resulting damage to cashew nut production. Implementing fire prevention measures or enhancing community awareness about fire safety practices becomes crucial in mitigating the risk of bushfire transmission in such scenarios. On the other hand, negative sensitivity indices indicate parameters where an increase leads to a decrease in model outcomes. For

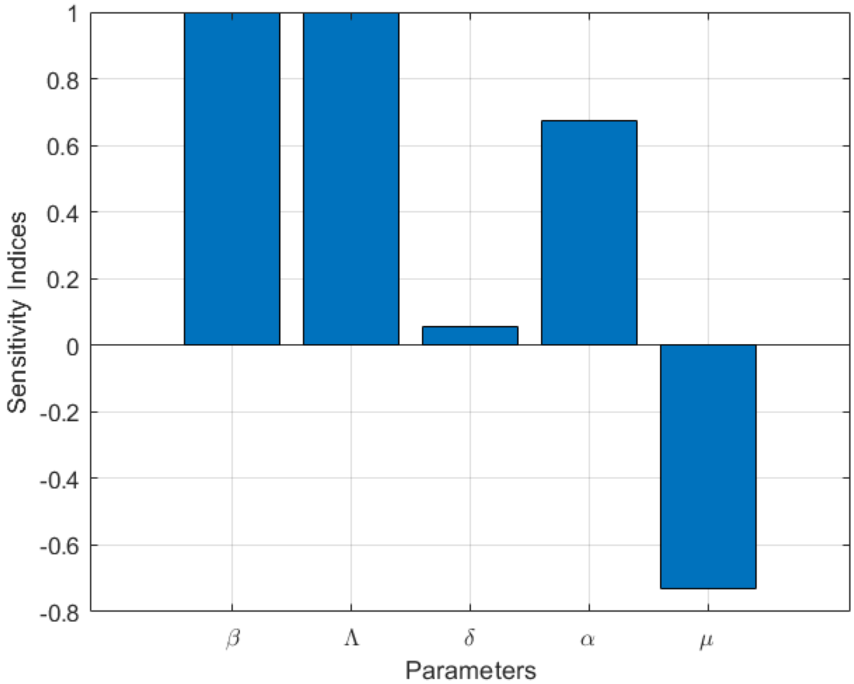

Figure 2: Sensitivity analysis of the basic fire-propagation number.

example, the negative sensitivity indices for natural death rate of cashew nut trees (μ) indicates that an increase in the natural death rate leads to a decrease in bushfire transmission dynamics and a reduction in the incidence of damage to cashew nut production. This implies that a higher natural death rate results in fewer susceptible trees available for fire propagation, thus reducing the overall spread of bushfires. Efforts to reduce the natural death rate, such as implementing tree maintenance practices or disease control measures, can significantly mitigate the impact of bushfires on cashew nut production.

4.4. Numerical Simulation

In this section, we implement the mathematical model proposed in the study into numerical algorithms using MATLAB software. This utilizes computational tools to simulate the intricate dynamics of bushfire transmission and its impact on cashew nut production. Parameter values from Table 1 are utilized as inputs for these simulations, ensuring a faithful reflection of real-world contexts. Additionally, initial conditions for the state variables, initialized as $S = 1000, I = 100, R = 0$, establish the simulation's starting point. These conditions offer a baseline scenario to observe the bush fires dynamics' evolution over time, capturing varying susceptibility levels, bush fire coverage, and bush fire prevalence for comprehensive analysis of the model's behaviour under different scenarios.

4.4.1. Overall Occurences of Bush Fires in the Population

The overall incidence of bushfires in the population represents the frequency of occurrences of bushfires observed within the entire demographic group studied, as outlined in the transmission dynamics model within this study's context. Figure 3 depicts the trend of bushfire occurrences over time in the entire population.

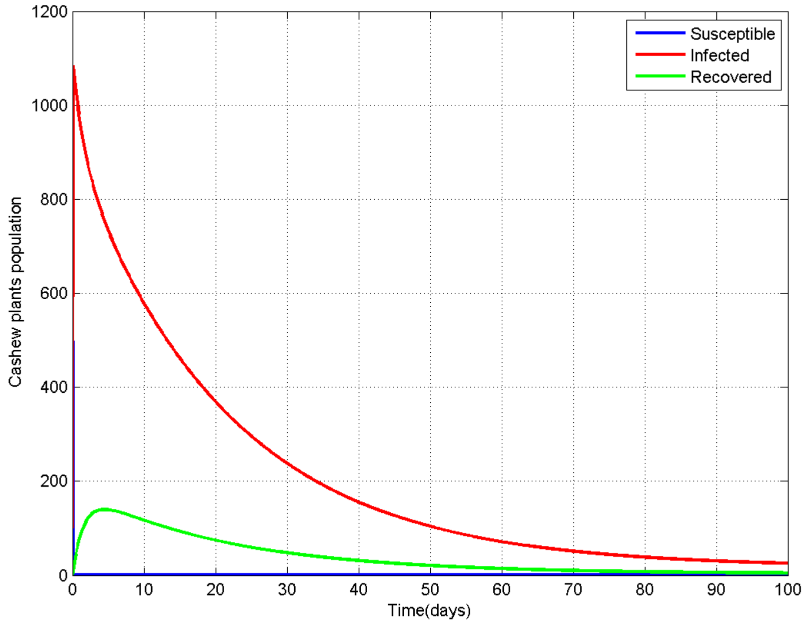

Figure 3: An Evolution of cashew plants population with time.

Figure 3 illustrates the time-based changes in the impact of bushfires on cashew nut production. Initially, prior to the occurrence of bush fires, the susceptible population of cashew nut trees is high, indicated by the elevated level of the blue line at the beginning of the simulation. This represents the healthy cashew nut trees before the onset of bush fires. As time progresses and bush fires occur, the number of susceptible cashew nut trees gradually decreases. The bush fires lead to the destruction of cashew nut trees, reducing the susceptible population over time. This decline in susceptible trees reflects the direct impact of bush fires on cashew nut production, as the trees become damaged or destroyed by the fires.

Concurrently, the number of affected cashew nut trees increases rapidly as bushfires spread, although the rate of increase diminishes with improved fire management practices. This reflects the spread of the impact of bush fires to neighboring trees, resulting in more trees becoming affected or damaged over time. The peak in the number of infected trees signifies the height of the bush fire impact on cashew nut production. Eventually, as the impact of bush fires subsides and recovery efforts take place, the number of recovered cashew nut trees gradually increases. The stabilization of the recovered population suggests the recovery of cashew nut production following the initial decline caused by bush fires.

4.5. Investigating the influence of parameter variations

This section conducts simulation by changing parameter values to examine the corresponding responses of state variables within the model framework. Particular attention is given to the parameters exerting the most significant influence.

4.5.1. Impact of bush fire propagation rates (β) on burnt trees

This involves investigating how variations in the speed at which fire spreads affect the number of trees that have been burned. This entails exploring how different rates of fire propagation influence the extent of damage sustained by trees within a particular environment. Figure 4 illustrates that with an increase in the bush fire propagation rate β,

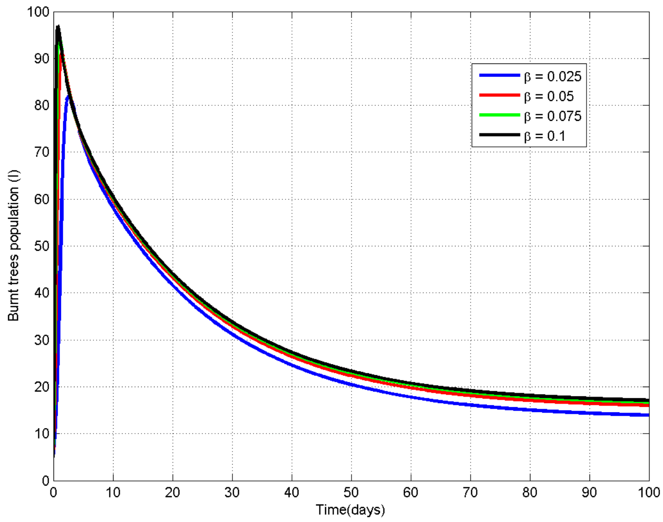

Figure 4: Impact of bush fire propagation rate on burnt trees.

there is a corresponding escalation in the rate of tree burnings. This acceleration results in a swifter spread of fires across the tree population. A higher bush fire propagation rate leads to a larger proportion of trees being affected by the fire. Consequently, the population of burnt trees may increase over time as the fire spreads rapidly. Implementing measures to reduce the transmission rate, such as establishing firebreaks or employing fire-resistant tree species, could be crucial in mitigating the extent of tree burnings.

4.5.2. Impact of variations in cashew trees natural death rate (μ) on infected cashew nut trees

This simulation explores the impact of varying the natural death rate of cashew nut trees (μ) on their susceptibility to bush fires infection. By adjusting μ, we aim to understand how the longevity of cashew nut trees influences their vulnerability to infection. The outcomes of this study provide valuable insights into the interplay between tree mortality rates and the dynamics of bushfire transmission, shedding light on the factors that shape the spread of fires within cashew nut tree populations. The results are presented in Figure 5. Figure 5 shows that as the natural death rate of cashew trees, μ, increases, it leads to

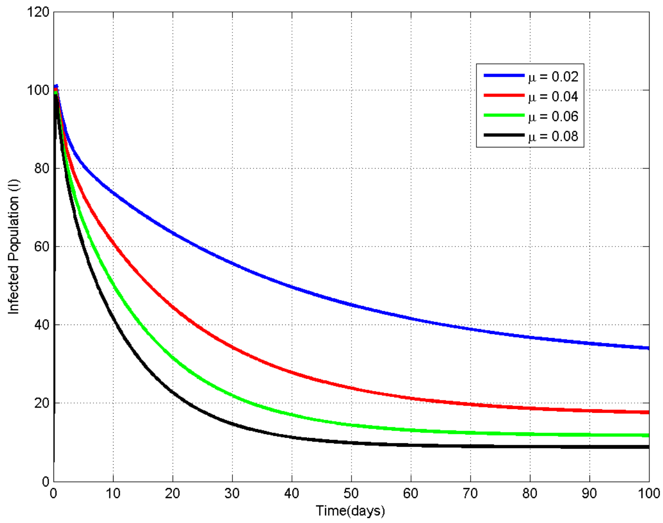

Figure 5: Impact of cashew trees natural death rate on burnt trees

a decrease in the population of susceptible cashew trees leading to a decrease of burnt cashew trees. This decrease occurs because a higher μ value results in a faster mortality rate among the cashew trees population. A lower population of susceptible cashew trees means there are fewer trees available to infect with fire incidence. Consequently, the rate of bush fire propagation from trees to trees decreases as μ increases. With fewer susceptible trees in the environment due to higher μ, susceptible cashew trees are less likely to encounter bush fire agents. Therefore, the overall health and infection of cashew trees may improve as μ increases. Reduced bush fire propagation and healthier cashew trees can lead to improved nut yield and quality. As the natural death rate of cashew trees increases, the likelihood of bush fire outbreaks decreases, resulting in better agricultural outcomes.

4.5.3. Impact of combined effect of β *and* μ *on* $\mathbb{R}_0$

In this section, we explore the combined impact of the parameters β and μ on the basic fire propagation number $\mathbb{R}_0$ in the context of bushfire dynamics affecting cashew nut production. By varying the bushfire propagation rate (β) and the natural death rate of susceptible cashew trees (μ), we aim to understand how changes in these parameters collectively influence the potential for fire spread within cashew nut tree populations. Through numerical simulations and graphical analysis, we will visualize how different combinations of β and μ values affect $\mathbb{R}_0$, providing insights into the conditions that promote or inhibit the spread of fires. The graphical representation in Figure 6 illustrates the results. Figure 6 illustrates that as the propagation rate of bushfires (β) increases, the $\mathbb{R}_0$ value tends to rise, indicating a heightened potential for fire spread within the cashew nut tree population. Conversely, lower β values result in diminished $\mathbb{R}_0$ values, indicating reduced fire transmission when the propagation rate is lower. Similarly, variations in the natural death rate of cashew trees (μ) also influence the $\mathbb{R}_0$ value. Higher

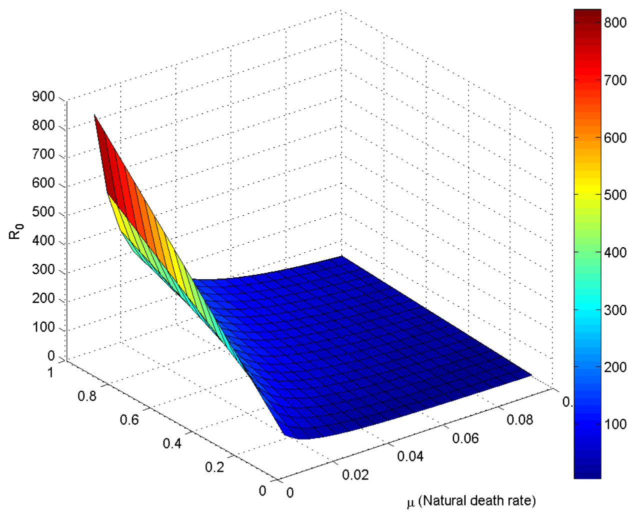

Figure 6: Impact of combined effect of β and μ on $\mathbb{R}_0$

μ values correspond to lower $\mathbb{R}_0$ values, suggesting that an increased natural death rate of cashew trees contributes to reduced fire spread, while lower μ values lead to increased fire transmission. The combined effect of both β and μ on $\mathbb{R}_0$ is revealed through the surface plot, where regions with higher elevations represent combinations of higher $\mathbb{R}_0$ values, indicative of conditions conducive to more rapid fire spread. Conversely, lower elevations signify combinations of lower $\mathbb{R}_0$ values, indicating conditions where fire transmission is less pronounced. Understanding these conditions can help farmers implement practices that suppress fire spread, such as maintaining firebreaks, reducing dry biomass, and ensuring proper forest management to create conditions less favorable for bushfire propagation.

5. Conclusion

In this study, we analyzed the transmission dynamics of bushfires and their impact on cashew nut production. We examined both positive and negative sensitivity indices to determine the relative importance of various parameters in influencing bushfire propagation and its consequences on cashew nut trees. Positive sensitivity indices indicated parameters that, when increased, led to proportional rises in bushfire transmission, while negative indices suggested decreases in model outcomes.

Our analysis revealed that parameters such as the replanting rate of cashew nut trees (Λ) and the effective contact rate (β) exhibited strong positive correlations with bushfire propagation, highlighting their significance in driving the epidemic dynamics. Conversely, parameters like the natural death rate of cashew nut trees (μ) showed negative correlations, indicating that increases in these rates could reduce bushfire transmission dynamics.

Through numerical simulations, we further explored the impact of parameter variations on bushfire spread and cashew nut production. By analyzing the basic fire propagation number ($\mathbb{R}_0$), we assessed the overall susceptibility of cashew nut populations to bushfires. Our findings underscored the importance of understanding the interplay between parameters like the bushfire propagation rate (β) and the natural death rate of cashew trees (μ) in shaping bushfire dynamics. The results which goes in line with the findings of the study conducted by Abdo et al. [37]

Our study offers actionable recommendations for mitigating bushfire risks in cashew nut production. Interventions targeting parameters identified as influential in our analysis, such as enhancing replanting efforts and reducing effective contact rates, can significantly curb bushfire transmission dynamics and minimize damage to cashew nut production.

Our findings contribute to the development of effective strategies for managing bushfire risks in cashew nut production systems. By implementing evidence-based control measures and leveraging insights from our study, stakeholders can mitigate the impact of bushfires on cashew nut production and safeguard agricultural livelihoods. These efforts align with the broader goals of sustainable development, particularly in ensuring food security and promoting economic prosperity in agricultural communities.

In future studies, incorporating additional factors such as weather patterns and land management practices could offer deeper insights into bushfire dynamics. Moreover, exploring the effectiveness of targeted interventions through scenario analyses may further refine strategies for mitigating bushfire risks in cashew nut production.

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Data Availability

All data used during this study are provided within this article in the list of references.

Conflicts of Interest

No conflict of interest to disclose.

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