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Numerical solution For HIV infection of CD4$^+$T cells using Taylor series method, Runge Kutta and Adams Bashforth method

ZAKIRULLAH[a,*]

[a] School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, 611731, China.

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Abstract

The HIV infection of CD4$^+$ T cells has been modeled by a system of first-order nonlinear differential equations. We applied the TSM, RK4, and ABM in this study. The model's numerical solution has been found in this work. The results show that, in comparison to the TSM approach and RK4, the relative error achieved by the ABM is smaller. When solving systems of nonlinear differential equations, the ABM is highly accurate.

Keywords: HIV Model, Taylor series method, Runge-Kutta method, Adams Bashforth method.

1. Introduction

Biology and mathematics are inextricably linked in various scientific fields[1]. Mathematical biology is beneficial to many fields, including genetics, environmental science, population dynamics, and medical research. Intricate biological phenomena may be conceptualized, understood, and visualized using mathematics. It is well known that the term "mathematical biology" refers to the study of biological phenomena or processes using mathematical methods, whereas the development of biological theory is referred to as theoretical biology. It suggests that using mathematics to analyze a biological system is necessary. Biologists use mathematics to develop models[2] that illustrate the necessary parts of the transmission mechanism of an illness when a study subject necessitates a method when using traditional lab methods is either impossible or too difficult. By using these techniques, researchers may "fine-tune" their study subjects while also predicting the likelihood of specific outcomes. Several axioms, laws, and assumptions that regulate these processes are used in these mathematical models to explain the complicated dynamics of biological events. The WHO estimates that 2.1 million persons got HIV globally in 2013,

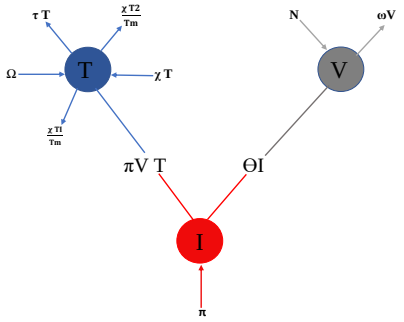
*Corresponding author: Zakirullahbzt@gmail.com

It estimates that the disease affected about 35 million people globally by the end of the year. Lentiviruses, of which HIV is a member, are known for their slow rates of action. Lentiviruses cause diseases that progress over a long period, disturbing the immune system in humans. HIV produces viral particles by converting viral RNA into DNA in the cell and then producing many copies of viral RNA. The virus has a chance of mutating, and each time it does, errors might result. After forming and infecting additional cells, these copies or virus particles affect the cell. CD4$^+$ T-cells are the primary target of HIV while attacking a wide range of cells; they play a key role in the body's general immune response to infections from the outside world. A growing case of acquired immune deficiency syndrome is caused by HIV (AIDS). It may take years for an HIV-positive person to acquire AIDS. Although there have been significant advances in medicine, there is currently no vaccine or effective treatment for HIV. Numerous mathematical models have been created since the early 1980s to better understand how HIV interacts with the human immune system and to test treatment strategies ([3],[4]). Silva and Torres developed a population model for the dynamics of TB-HIV/AIDS coinfection transmission to examine the best strategies for reducing the number of people with TB and AIDS. In their study, Wang et al. look at an HIV model with latent infection and antiretroviral treatment. We recommend the reader read the excellent review article on [6] that discusses HIV mathematical modeling of several phenomena. Through quantitative analysis of HIV replication in vivo, our understanding of the pathogenesis of AIDS and the efficiency of antiretroviral treatment has significantly improved [7]. We provide references for a comprehensive mathematical analysis of such models [8].
For the purpose of this study, we used the RK4, TSM, and ABM to solve systems of nonlinear differential equations([9] ,[10]). With the TSM, RK4, and ABM methods, the numerical solution of the HIV model is to be established, as well as ways to compare these methods. The ABM is the search for the value function y (x) at point x in a set of nonlinear ordinary differential equations of first order with the starting value $y(x_0) = y_0$, the RK4 method is mostly used to obtain the initial values for ABM.

2. Structure of HIV Dynamics

In order to formulate a realistic model of T-cell infection by HIV, we will now analyze the population dynamics of T cells in the absence of HIV. In the bone marrow, T cells and other lymphocytes are produced. The differentiation and maturation of immature T cells in the thymus results in the production of immune-competent T cells. The involutionary phenomena, which affects the thymus, is characterized by a loss of weight and volume as well as microscopic signs of degradation. Human puberty is the period when the thymus reaches its maximal weight and begins to gradually involute. Some of the lymphocytes in the adult thymus still act as immunocompetent T cells and T-cell progenitors, excision of the thymus frequently has minimal impact. This model's primary goal is to understand CD4 + T cells. It is essential to understand how HIV interacts with CD4 + T cells in order to understand HIV/AIDS infections. Several studies have shown that these cells are created throughout the bone marrow before being transported to the medulla and going through a specific differentiation process to develop into CD4 + T cells that are free of infection. The thymus in the human body carries the most weight. As to growing increasingly complex,

the human thymus reaches its maximum weight during the maturation phase. Although the adult thymus is active, it's few lymphocytes serve as receptors for T-cells, both infected and uninfected, and thymic drainage has a small effect on adults. The number of CD4+ T cells can be utilized to estimate how persistent an HIV infection is since they provide more information on early symptoms. The oscillatory and path-tracking behavior of the HIV dynamic is being examined in this study. By focusing on the essential components, these studies assist policymakers in identifying input factors for infection prevention.
The following compartmental diagram 1 explains the scenario well;



$$\begin{aligned}
\frac{dT}{dt} &= \Omega - \tau T + \chi T\left(1 - \frac{T+I}{T_m}\right) - \pi V T,\\
\frac{dI}{dt} &= \pi V T - \theta I,\\
\frac{dV}{dt} &= N\theta I - \omega V
\end{aligned} \tag{2.1}$$

with the initial conditions

where the blood's concentrations of susceptible CD4^{+}T cells, infected CD4^{+}T cells, and free HIV viral particles are represented by T (t), I (t), and V (t) respectively. In this paper, we used $\Omega = 0.1$, $\tau = 0.02$, $\Theta = 0.3$, $\chi = 3$, $\omega = 2.4$, $\pi = 0.0027$, $T_m = 1500$, and

$N = 10$. The parameters τ and Ω stand for the normal turnover rates of virus particles, uninfected T cells, and infected T cells, respectively. The logistic growth of the healthy CD4$^+$T cells is now described by $\chi T(1 - \frac{T-I}{T_m})$, whereas the growth of infected CD4$^+$T cells is disregarded. The term "π VT" refers to the prevalence of HIV infection in healthy CD4$^+$T cells, where "$\pi > 0$" is the infection rate. It is assumed that each infected C4$^+$T cell, including any daughter cells, produces N virus particles over the course of its life. It is assumed that the body produces CD4$^+$T cells at a constant rate from bone marrow and thymus precursors. When stimulated by an antigen or mitogen, T cells undergo mitosis at a rate of χ, and T_m is the maximum level of CD4$^+$T cell concentration in the body.

3. Taylor series method

In this section, the general solution and its numerical solution of the proposed HIV model 2.1 will be examined in this section using the TSM, we will perform some steps: First of all we compute the first derivative of the $T(t), I(t) and V(t)$ as

$$\begin{aligned} T'(t_0) &= \Omega - \tau T_0 + \chi T_0 \left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0, \\ I'(t_0) &= \pi V_0 T_0 - \theta I_0, \\ V'(t_0) &= N\theta I_0 - \omega V_o. \end{aligned} \tag{3.1}$$

We compute the second derivative of T(t), I(t) and V(t) as

$$\begin{aligned} T''(t_0) &= (\Omega - \tau T_0 + \chi T_0 \left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0)(\chi - \tau - \pi V_0 - \frac{2\chi T_0}{T_m} - \frac{\chi I_0}{T_m}) \\ &- \frac{\chi T_0}{T_m}(\pi V_0 T_0 - \theta I_0) - \pi T_0(N\theta I_0 - \omega V_0) \\ I''(t_0) &= \pi V_0(\Omega - \tau T_0 + \chi T_0 \left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0) - \theta(\pi V_0 T_0 - \theta I_0) + \pi T_0(N\theta I_0 - \omega V_0) \\ V''(t_0) &= N\theta(\pi V_0 T_0 - \theta I_0) - \omega(N\theta I_0 - \omega V_0) \end{aligned} \tag{3.2}$$

We compute the 3rd derivative of T(t), I(t) and V(t) as

$$\begin{aligned} T'''(t_0) &= (\Omega - \tau T_0 + \chi T_0 \left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0)(I_0^2 + \pi^2 V_0^2 + \frac{2\chi^2 T_0 I_0}{T_m^2} - \frac{4\pi\chi V_0 T_0}{T_m} \\ &+ \frac{2\pi\chi T_0 V_0^2}{T_m} - \pi\chi I_0 V_0 - \pi\chi I_0 V_0 + \frac{2\chi^2 I_0}{T_m} - \frac{\tau I_0 \chi}{T_m} + 2\pi\tau V_0 - 2\pi\chi V_0) \\ &+ (\pi V_0 T_0 - \theta I_0)(\frac{\Omega\chi}{T_m} + 2T_0 I_0 + \frac{\chi^2 T_0^2 I_0}{T_m^2} - \pi\tau T_0 V_0 + \frac{2\chi^2 T_0}{T_m} - \frac{2\tau\chi T_0}{T_m}) \\ I'''(t_0) &= (\Omega - \tau T_0 + \chi T_0 \left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0)(\pi\theta N I_0 - \pi^2 V_0^2 - \pi\omega - \pi\tau - \pi\theta T_0 - \frac{\pi\chi T_0 I_0}{T_m}) \\ &+ (\pi V_0 T_0 - \theta I_0)(\theta^2 + \pi\theta N T_0 - \frac{\pi\chi T_0 V_0}{T_m}) + (N\theta I_0 - \omega V_0)(\pi\Omega - 2\pi^2 V_0 T_0 - \pi\omega T_0 - \pi\tau T_0 \\ &- \pi\theta T_0 - \frac{\pi\chi T_0 I_0}{T_m}) \end{aligned} \tag{3.3}$$

$$V'''(t_0) = N\pi\theta V_0(\Omega - \tau T_0 + \chi T_0\left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0) - (\pi V_0 T_0 - \theta I_0)(\theta^2 N - \omega\theta N) + (\pi\theta N T_0 + \omega^2)(N\theta I_0 - \omega V_0)$$

Now the solution for the first few terms is now given by

$$\begin{aligned} T(t) &= T(t_0) + T'(t_0)h + T''(t_0)\frac{h^2}{2!} + T'''(t_0)\frac{h^3}{3!} + \cdots, \\ I(t) &= I(t_0) + I'(t_0)h + I''(t_0)\frac{h^2}{2!} + I'''(t_0)\frac{h^3}{3!} + \cdots, \\ V(t) &= V(t_0) + V'(t_0)h + V''(t_0)\frac{h^2}{2!} + V'''(t_0)\frac{h^3}{3!} + \cdots. \end{aligned} \tag{3.4}$$

Substituting the values of equations (3.1), (3.2) and (3.3) in (3.4), we obtain

$$\begin{aligned} T(t) = T(t_0) &+ \left(\Omega - \tau T_0 + \chi T_0\left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0\right)h + \left((\Omega - \tau T_0 + \chi T_0\left(1 - \frac{T_0 + I_0}{T_m}\right)\right. \\ &- \pi V_0 T_0)(\chi - \tau - \pi V_0 - \frac{2\chi T_0}{T_m} - \frac{\chi I_0}{T_m}) - \frac{\chi T_0}{T_m}(\pi V_0 T_0 - \theta I_0) - \pi T_0(N\theta I_0 - \omega V_0)\Big)\frac{h^2}{2!} \\ &+ \left((\Omega - \tau T + \chi T\left(1 - \frac{T + I}{T_m}\right) - \pi V T)(I^2 + \pi^2 V^2 + \frac{2\chi^2 TI}{T_m^2} - \frac{4\pi\chi VT}{T_m} + \frac{2\pi\chi TV^2}{T_m}\right. \\ &- \pi\chi IV - \pi\chi IV + \frac{2\chi^2 I}{T_m} - \frac{\tau I\chi}{T_m} + 2\pi\tau V - 2\pi\chi V) + (\pi VT - \theta I)(\frac{\Omega\chi}{T_m} + 2TI + \frac{\chi^2 T^2 I}{T_m^2} \\ &\left.- \pi\tau TV + \frac{2\chi^2 T}{T_m} - \frac{2\tau\chi T}{T_m})\right)\frac{h^3}{3!} + \cdots, \\ I(t) = I(t_0) &+ \left(\pi V_0 T_0 - \theta I_0\right)h + \left(\pi V_0(\Omega - \tau T_0 + \chi T_0\left(1 - \frac{T_0 + I_0}{T_m}\right) - \pi V_0 T_0) - \theta(\pi V_0 T_0\right. \\ &\left.- \theta I_0) + \pi T_0(N\theta I_0 - \omega V_0)\right)\frac{h^2}{2!} + \left((\Omega - \tau T + \chi T\left(1 - \frac{T + I}{T_m}\right) - \pi V T)(\pi\theta N I - \pi^2 V^2\right. \\ &\left.- \pi\omega - \pi\tau - \pi\theta T - \frac{\pi\chi TI}{T_M}) + (\pi VT - \theta I)(\theta^2 + \pi\theta NT - \frac{\pi\chi TV}{T_m}) + (N\theta I - \omega V)(\pi\Omega\right. \\ &\left.- 2\pi^2 VT\pi\omega T - \pi\tau T - \pi\theta T - \frac{\pi\chi TI}{T_{T_m}})\right)\frac{h^3}{3!} + \cdots, \\ V(t) = V(t_0) &+ \left(N\theta I_0 - \omega V_0\right)h + \left(N\theta(\pi V_0 T_0 - \theta I_0) - \omega(N\theta I_0 - \omega V_0)\right)\frac{h^2}{2!} \\ &+ \left(N\pi\theta V(\Omega - \tau T + \chi T\left(1 - \frac{T + I}{T_m}\right) - \pi VT) - (\pi VT - \theta I)(\theta^2 N - \omega\theta N) + (\pi\theta NT\right. \\ &\left.+ \omega^2)(N\theta I - \omega V)\right)\frac{h^3}{3!} + \cdots. \end{aligned} \tag{3.5}$$

After putting the numerical values, we obtained the following results.
$T(0.2) = 0.7945672416$
$I(0.2) = 0.0134256843626$

$V(0.2) = 0.5954362745$
Solutions T(t), I(t) and V(t) can be solved in the same manner as above.

Table 1: Solution TSM

t	T(t)	I(t)	V(t)
0	0.1	0	0.1
0.2	0.3405672416	0.0000138102143	0.05989984991
0.4	0.5242405113	0.0000221004762	0.04382941178
0.6	0.9645736342	0.0000401774193	0.03890455014
0.8	1.4538941316	0.0000491315834	0.01327378342
1	2.0258634211	0.0000693270214	0.01223876351

4. Runge-Kutta method

The RK4 is a numerical method that is one-step because this method requires only one previous point to compute the new value. The RK4 is often used; the Runge-Kutta method of order four, it is the most stringent. As a result, while solving a differential equation, the RK4 is often applied.
Formulations of the RK4 (2.1)

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{4.1}$$

with

$$k_1 = h(x_i, y_i, z_i)$$
$$k_2 = h(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}, z_i + l_1 + \frac{h}{2})$$
$$k_3 = h(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}, z_i + \frac{l_2}{2})$$
$$k_4 = h(x_i + h, y_i + k_3, z_i + l_3)$$

This method is more accurate, is simple to develop, is stable, and has small cut and rounding errors.The HIV model of cell infection CD4$^+$T shown in equations will be further identified in the form of ordinary differential equations as shown in equations(2.1).

$$\frac{dT}{dt} = \phi(t, T, I, \mathcal{V}) = \Omega - \alpha T + \tau T(1 - \frac{T + I}{T_m}) - \pi \mathcal{V} T$$

$$\frac{dI}{dt} = \varphi(t, T, I, \mathcal{V}) = \pi \mathcal{V} T - \theta I \tag{4.2}$$

$$\frac{d\mathcal{V}}{dt} = \psi(t, T, I, \mathcal{V}) = N\theta I - \omega \mathcal{V}$$

The equation (4.2) above is solved using the RK4 based on equation (2.1) by substitution, so that the following equation (4.3) is obtained:

$$\begin{aligned} T_{r+1} &= T_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \\ I_{r+1} &= I_r + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)h \\ V_{r+1} &= V_r + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h \end{aligned} \tag{4.3}$$

with is the step of time

$$\begin{aligned} k_1 &= \Omega - \alpha T_r + \tau T_r(1 - \frac{T_r + I_r}{T_m}) - \pi V_r T_r \\ k_2 &= \Omega - \alpha(T_r + \frac{h}{2}) + \tau(T_r + \frac{h}{2})(1 - \frac{(T_r + \frac{h}{2}) + (I_r + \frac{k_1}{2})}{T_m}) \\ &\quad - \pi(V_r + \frac{l_1}{2})(T_r + \frac{h}{2}) \\ k_3 &= \Omega - \alpha(T_r + \frac{h}{2}) + \tau(T_r + \frac{h}{2})(1 - \frac{(T_r + \frac{h}{2}) + (I_r + \frac{k_2}{2})}{T_M}) \\ &\quad - \pi(V_r + \frac{l_2}{2})(T_r + \frac{h}{2}) \\ k_4 &= \Omega - \alpha(T_r + h) + \tau(T_r + h)(1 - \frac{(T_r + h) + (I_r + k_3)}{T_m}) \\ &\quad - \pi(V_r + l_3)(T_r + h) \end{aligned} \tag{4.4}$$

$$\begin{aligned} l_1 &= \pi V_r T_r - \theta I_r \\ l_2 &= \pi(V_r + \frac{l_1}{2})(T_r + \frac{h}{2}) - \theta(I_r + \frac{k_1}{2}) \\ l_3 &= \pi(V_r + \frac{l_2}{2})(T_r + \frac{h}{2}) - \theta(I_r + \frac{k_2}{2}) \\ l_4 &= \pi(V_r + l_3)(T_r + h) - \theta(I_r + k_3) \end{aligned} \tag{4.5}$$

$$
\begin{aligned}
m_1 &= N\theta I_r - \omega V_r \\
m_2 &= N\theta(I_r + \frac{k_1}{2}) - \omega(V_r + \frac{l_1}{2}) \\
m_3 &= N\theta(I_r + \frac{k_2}{2}) - \omega(V_r + \frac{l_2}{2}) \\
m_4 &= N\theta(I_r + k_3) - \omega(V_r + l_3)
\end{aligned}
\tag{4.6}
$$

equation (4.3)is a numerical solution of the HIV infection of CD4$^+$T cells. Substituting (4.4),(4.5) and (4.6) into (4.3) result in

$$
\begin{aligned}
T_{r+1} = T_r + \frac{1}{6}h\Big(& \Omega - \alpha T_r + \tau T_r(1 - \frac{T_r + I_r}{T_m}) - \pi V_r T_r + 2(\Omega - \alpha(T_r + \frac{h}{2}) + \tau(T_r + \frac{h}{2}) \\
& (1 - \frac{(T_r + \frac{h}{2}) + (I_r + \frac{k_1}{2})}{T_m}) - \pi(V_r + \frac{l_1}{2})(T_r + \frac{h}{2})) + 2(\Omega - \alpha(T_r + \frac{h}{2}) + \tau(T_r + \frac{h}{2}) \\
& (1 - \frac{(T_r + \frac{h}{2}) + (I_r + \frac{k_2}{2})}{T_m}) - \pi(V_r + \frac{l_2}{2})(T_r + \frac{h}{2})) + \Omega - \alpha(T_r + h) + \tau(T_r + h) \\
& (1 - \frac{(T_r + h) + (I_r + k_3)}{T_m}) - \pi(V_r + l_3)(T_r + h)\Big)
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
I_{r+1} = I_r + \frac{1}{6}h\Big(& \pi V_r T_r - \theta I_r + 2(\pi(V_r + \frac{l_1}{2})(T_r + \frac{h}{2}) - \theta(I_r + \frac{k_1}{2})) + 2(N\theta(I_r + \frac{k_2}{2}) \\
& - \omega(V_r + \frac{l_2}{2})) + \pi(V_r + l_3)(T_r + h) - \theta(I_r + k_3)\Big)
\end{aligned}
\tag{4.8}
$$

$$
\begin{aligned}
V_{r+1} = V_r + \frac{1}{6}h\Big(& N\theta I_r - \omega V_r + 2(N\theta(I_r + \frac{k_1}{2}) - \omega(V_r + \frac{l_1}{2}) + 2(N\theta(I_r + \frac{k_2}{2}) \\
& - \omega(V_r + \frac{l_2}{2})) + N\theta(I_r + k_3) - \omega(V_r + l_3)\Big)
\end{aligned}
\tag{4.9}
$$

Using the initial condition T_0, I_0, V_0 into equations (4.2) ,(4.3) and (4.4) we have

$$
\begin{aligned}
T_1 = T_0 + \frac{1}{6}h\Big(& \Omega - \alpha T_0 + \tau T_0(1 - \frac{T_0 + I_0}{T_m}) - \pi V_0 T_0 + 2(\Omega - \alpha(T_0 + \frac{h}{2}) + \tau(T_0 + \frac{h}{2}) \\
& (1 - \frac{(T_0 + \frac{h}{2}) + (I_0 + \frac{k_1}{2})}{T_m}) - \pi(V_0 + \frac{l_1}{2})(T_0 + \frac{h}{2})) + 2(\Omega - \alpha(T_0 + \frac{h}{2}) \\
& + \tau(T_0 + \frac{h}{2})(1 - \frac{(T_0 + \frac{h}{2}) + (I_0 + \frac{k_2}{2})}{T_m}) - \pi(V_0 + \frac{l_2}{2})(T_0 + \frac{h}{2})) + \Omega - \alpha(T_0 + h) \\
& + \tau(T_0 + h)(1 - \frac{(T_0 + h) + (I_0 + k_3)}{T_m}) - \pi(V_0 + l_3)(T_0 + h)\Big)
\end{aligned}
\tag{4.10}
$$

$$I_1 = I_0 + \frac{1}{6}h\Big(\pi V_0 T_0 - \theta I_0 + 2(\pi(V_0 + \frac{l_1}{2})(T_0 + \frac{h}{2}) - \theta(I_0 + \frac{k_1}{2})) + 2(N\theta(I_0 + \frac{k_2}{2}) - \omega(V_0 + \frac{l_2}{2})) + \pi(V_0 + l_3)(T_0 + h) - \theta(I_0 + k_3)\Big) \tag{4.11}$$

$$V_1 = V_0 + \frac{1}{6}h\Big(N\theta I_0 - \omega V_0 + 2(N\theta(I_0 + \frac{k_1}{2}) - \omega(V_0 + \frac{l_1}{2}) + 2(N\theta(I_0 + \frac{k_2}{2}) - \omega(V_0 + \frac{l_2}{2})) + N\theta(I_0 + k_3) - \omega(V_0 + l_3)\Big) \tag{4.12}$$

Plug $T_0 = 0.1, \quad I_0 = 0, \quad V_0 = 0.1 \quad and \quad h = 0.2$ in equations (4.5), (4.6), (4.7) for the three-component model. A first approximations for T (t), I(t), and V(t) are calculated and presented below:

$T(0.2) = 0.2088080833$
$I(0.2) = 0.0000212400378$
$V(0.2) = 0.06187984331$

Solutions With the help of the Matlab software, T(t), I(t), and V(t) may be solved in the same way as above after the third iteration until the first calculation is performed in order to obtain a system solution equation (2.1) using the RK4 method as shown in table 3

Table 2: Solution RK4

t	T(t)	I(t)	V(t)
0	0.1	0	0.1
0.2	0.208808083	0.0000212400378	0.06587984331
0.4	0.406240539	0.0000330177401	0.05482948878
0.6	0.764423889	0.0000560177419	0.04370455014
0.8	1.314046830	0.0000610173545	0.02112036322
1	1.691594802	0.0000834256843	0.02100845073

5. Adams Bashforth Moulton method

Finding the value function y (x) at point x in specific first-order nonlinear ordinary differential equations $y' = f(x, y)$, as well as the initial value $y(x_0) = y_0$ that is known to predict the predictor equation, and then correcting the equation corrector, is required in the ABM. The one-step approach can be used to get the initial necessary values for the ABM. The fourth-order RK4 and the ABM can be combined to provide excellent results.

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{5.1}$$

Corrector ABM formula

$$y_{r+1} = y_r + \frac{h}{24}(9py'_{n+1} + 19y'_n - 5y_{n-1} + y'_{n-2}) \tag{5.2}$$

Calculating four initial solution $T_{0...3}$, $I_{0...3}$ and $V_{0...3}$ using Runge Kutta 4^{th} Order method. Determining the value of $\phi_n, \phi_{n-1}, \phi_{n-2} and \phi_{n-3} with n = 3, 4, ... defined as follows$:

$$\begin{aligned}\phi_{n-3} = \phi_0 &= \phi(t_0, T_0, I_0, V_0)\\ \phi_{n-2} = \phi_1 &= f(t_1, T_1, I_1, V_1)\\ \phi_{n-1} = \phi_2 &= f(t_2, T_2, I_2, V_2)\\ \phi_n = \phi_3 &= f(t_3, T_3, I_3, V_3)\end{aligned}$$

For value ξ_n and ψ_n do the steps as in ϕ_n Calculation of predictor ABM4^{th} Order: $pT_{n+1} = T_n + \frac{h}{24}(55\phi_n - 59\phi_{n-1} + 37\phi_{n-2} + 9\phi_{n-3})$

$pT_{3+1} = T_3 + \frac{h}{24}(55\phi_3 - 59\phi_2 + 37\phi_1 + 9\phi_0)$

$pT_4 = 1.39954619$
Calculation of correcctor ABM 4^{th} Order: $T_{n+1} = T_n + \frac{h}{24}(9\phi_{n+1} + 19\phi_n - 5\phi_{n-1} + \phi_{n-2})$

$T_4 = T_n + \frac{h}{24}(9\phi_4 + 19\phi_3 - 5\phi_2 + \phi_1)$

$T_4 = 1.3991351404$
Similarly,
For value ξ_n and ψ_n do the steps as in ϕ_n Calculation of predictor ABM4^{th} Order: $pI_{n+1} = I_n + \frac{h}{24}(55\phi_n - 59\phi_{n-1} + 37\phi_{n-2} + 9\phi_{n-3})$

$pI_{3+1} = I_3 + \frac{h}{24}(55\phi_3 - 59\phi_2 + 37\phi_1 + 9\phi_0)$

$pI_4 = 0.00002011624315$
Calculation of correcctor ABM 4^{th} Order: $I_{n+1} = I_n + \frac{h}{24}(9\phi_{n+1} + 19\phi_n - 5\phi_{n-1} + \phi_{n-2})$

$I_4 = I_n + \frac{h}{24}(9\phi_4 + 19\phi_3 - 5\phi_2 + \phi_1)$

$I_4 = 0.0000201004115$
Solutions With the help of the Matlab software, T(t), I(t), and V(t) may be solved in the same way as above after the third iteration until the first calculation is performed in order to obtain the solution of equation (2.1) using the ABM as shown in table 3.
For value ξ_n and ψ_n do the steps as in ϕ_n Calculation of predictor ABM4^{th} Order: $pV_{n+1} = V_n + \frac{h}{24}(55\phi_n - 59\phi_{n-1} + 37\phi_{n-2} + 9\phi_{n-3})$

$pV_{3+1} = V_3 + \frac{h}{24}(55\phi_3 - 59\phi_2 + 37\phi_1 + 9\phi_0)$

$pV_4 = 0.004181112311$
Calculation of correcctor ABM 4^{th} Order: $V_{n+1} = V_n + \frac{h}{24}(9\phi_{n+1} + 19\phi_n - 5\phi_{n-1} + \phi_{n-2})$

$V_4 = V_n + \frac{h}{24}(9\phi_4 + 19\phi_3 - 5\phi_2 + \phi_1)$

$V_4 = 0.004180010311$

Table 3: T(t)

t	Predictor values of V	Corrector values of V	ABM
0	-	-	0.1
0.2	–	–	0.2088080833
0.4	–	–	0.4062405393
0.6	–	–	0.764423889
0.8	1.20985643012	1.20954619	1.20954619
1	1.46205159839	1.46204156	1.462041576

Table 4: I(t)

t	Predictor values of I	Corrector values of I	ABM
0	-	-	0.1
0.2	–	–	0.0000212400378
0.3	–	–	0.0000330177401
0.6	–	–	0.0000560177419
0.8	0.0000801688417	0.0000801523407	0.0000801523407
1	0.0000923393215	0.0000923270214	0.0000923270214

Table 5: V(t)

t	Predictor values of V	Corrector values of V	ABM
0	-	-	0.1
0.2	-	-	0.06587984331
0.4	-	-	0.05482948878
0.6	-	-	0.04370455014
0.8	0.03115213142	0.03110010311	0.03110010311
1	0.0301586923	0.0300845043	0.0300845043

Table 6: Description of parameters and its values with the initial conditions used for simulation

Parameters	Parameter Interpretation	Numerical value	Source
$T(t=0)=T_0$	Initial Condition of Susceptible Cells.	0.1	[12]
$I(t=0)=I_0$	Initial Condition of Infected Cells.	0	[12]
$V(t=0)=V_0$	Initial Condition of Infection–Free Cells.	0.1	[12]
τ	Death rate of healthy T-cells	0.02 day^{-1}	[12]
θ	Death rate of infected T-cells	0.3 day^{-1}	[12]
ω	Death rate of HIV virus	2.4 day^{-1}	[12]
π	Infection rate of T-cells by free virus	0.0027 day^{-1}	[12]
Ω	Healthy T-cells supply rate from precursors	0.1 mm^{-3}	[12]
χ	Growth rate of healthy T-cells	3 day^{-1}	[12]
T_M	Maximum number of healthy T-cells	1500 mm^{-3}	[12]
N	The quantity of virus generated by infected T-cells	10 day^{-1}	[12]

6. MATLAB Software for HIV Model Simulation:

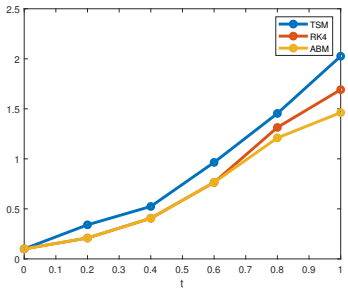


Figure 1: T(t)

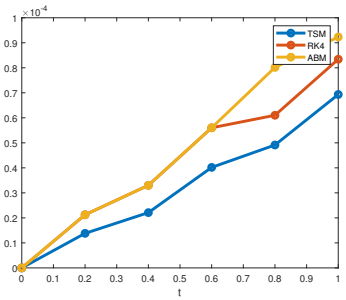


Figure 2: I(t)

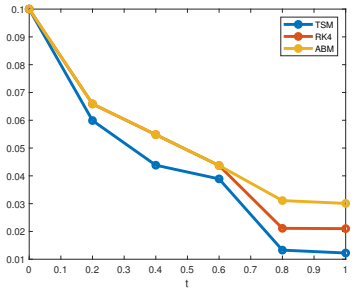


Figure 3: V(t)

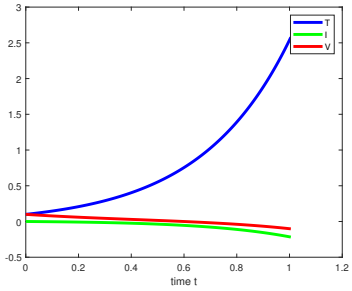


Figure 4:

1ABM solutions of T(t) for six iterations are compared . 2 ABM solutions of I(t) for six iterations are compared. 3 ABM solutions of V(t) for six iterations are compared. 4 The ABM solutions of 2.1 for six iterates.

7. Conclusion

Based on the discussion's conclusions, it is possible to reach the conclusion that the ABM may be used to solve the system of nonlinear differential equations in the HIV model infection of CD4$^+$T cells in steps similar to those described above. We observed that the ABM is more accurate and that the approximate solution converged to the exact solution quickly when comparing the results of the two methods. The ABM is powerful and more efficient at solving the initial value problem numerically (IVP).

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