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Taylor Series Expansion Method To Compute Approximate Solution for Nonlinear Dynamical System

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Abstract

In this manuscript we have studied a five compartmental mathematical model of Ebola epidemic. The suggested mathematical model is classified into susceptible, incubation, infected, isolated infected and recovered classes. The Taylor series method (TSM) is used to achieve the approximate results for each compartment. The graphical presentation that corresponds to some real facts is given.

Keywords: Ebola virus, Approximate solution, TSM.
2010 MSC: 47E05, 92D30, 34A45.

1. Introduction

Ebola Virus Disease (EVD) is a devastating disease that affects humans and nonhuman primates(monkeys, gorillas, and chimpanzees). Ebola epidemic occurred in 2014 which brought a severe economic hardship to the people of west Asia [1, 2]. It was formally restricted to central part of Africa but recently the epidemic had occurred in the western part of Africa. While W.H.O reported the total number of cases in west Africa in 2014 [3, 4, 5]. However, Liberia also noted the highest number of cases. The epidemiological patient is characterized various symptoms in first stage, such as Headache, Malaise fever abdominal pain and asthenia [6, 7]. While in the second stage of infection is after a week of inception, skin rash, usually show up, kidney impaired and lever function, which can bring 50 percent to 90 percent death chances within ten days. Although new vaccinations and therapies are being investigated, there are no licensed medications or vaccines to treat EVD. Recovery appears to be influenced by the amount of virus a person was exposed to at the outset, the timing of therapy, and the patient's age and immune response. Early supportive care, such as maintaining bodily fluids and electrolytes and monitoring blood pressure, can enhance survival chances by giving the body's immune system enough time to fight off the infection. Younger folks appear to recuperate more quickly than their

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elders. Those who survive acquire antibodies that can last for up to ten years. Long-term consequences, including as joint and eye impairments, affect some survivors.

Here we remark that infectious diseases are treating via different tools and procedures. One of the important tool is mathematical modeling of infectious disease. Mathematical models can help us about to investigate the transmission, controlling and eradication of the infectious in our society. The idea of mathematical model was first built by Bernoulli in 1776. The first formal mathematical model was formulated by Mckendrick and Karmark in 1927, who presented a simple model known as SIR (susceptible, infected and recovered). After their idea has been extended to form various mathematical models of different disease in humans as well as animals, plants, etc. Therefore in recent times, mathematical models in epidemiology present a strong framework in understanding the mechanism of various disease. On the other hand, mathematical models also can help us in understanding and to construct some strategies, how to control the disease from being spreading and to take precautionary necessary measures.

Here it is worth mentioning that researchers study mathematical models from different aspects including qualitative theory, numerical and optimization analysis, etc. In same fashion, some authors have investigated the mathematical model which is describing the dynamic of Ebola disease and its optimal control to examine vaccination effect on the disease. Keeping in mind the treatment of the epidemic models, researchers have investigated the current disease from different aspects[8, 9, 10, 11]. We refer here to some recent models related to the topic of research, see [12, 13, 14, 15, 16, 17, 18, 19, 20]. Here we investigate the proposed model for approximate solution by using Taylor series expansion method. This method is a powerful tool from which the other basic numerical methods like, Euler, modified Euler method, Heun method etc are derived. Further the concerned numerical results are displayed graphically by using some real values of the parameters and initial population. By using mathematical model of Ebola virus disease, we can easily understand the transmission mechanism in society.

2. Preliminaries

To obtain these results we need basic definition of Taylor series, given as

Definition 2.1. [21] If a function $f(x)$ is such that $f(x), f'(x), f''(x), f'''(x), ..., f^{n-1}(x)$ are said to be continuous on the closed interval $[x, x+h]$ and $f^{n}(x)$ exist in the open interval $(x, x+h)$ then there exist a real under θ between 0 and 1 such that
$f(x+h) = f(x) + h\, f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + ... \frac{h^n}{n!} f^{n}(x+\theta h)$.

3. Formulation of the Proposed Model

We present the Ebola epidemic $SEII_hR$ model proposed by [22], where $S(t)$ is the number of susceptible humans at time t; $E(t)$ is the incubation period with which the disease manifests at time t; $I(t)$ is the number of infected humans in the population at time t; $I_h(t)$ represents the isolation infectious human population compartment at t and $R(t)$ is the number of infected humans who recovered at time t. The entire human population is denoted by $N(t) = S(t) + E(t) + I(t) + I_h(t) + R(t)$. The parameter μ deals with the death rate and β has to do with the transmission rate of the disease from infectious person to susceptible. The recruitment rate is denoted by b. Similarly, $1/\delta$ and $1/\gamma$ are durations of stay in the compartments of $E(t)$ and $I(t)$ respectively. The duration an infected patient transfer from isolation compartment to death is represented by $1/\omega$. The rate of proportion of exposed and infected humans move into isolated compartment is denoted by λ and α respectively. Due to interactions of infected humans with virgin population, we obtain the following system of five nonlinear differential equations proposed in [22].

$$\begin{cases} \frac{dS(t)}{dt} = \mu N - \frac{\beta(t)SI}{N} - \mu S, \\ \frac{dE(t)}{dt} = \frac{\beta(t)SI}{N} - (\mu+\delta+\lambda)E, \\ \frac{dI(t)}{dt} = \delta E - (\gamma+\alpha+\mu)I, \\ \frac{dI_h(t)}{dt} = \lambda E + \alpha I - (\omega+\mu)I_h, \\ \frac{dR(t)}{dt} = \gamma I + \omega\ H - \mu R, \end{cases} \tag{3.1}$$

with given initial conditions
$S(0) = S_0,\ E(0) = E_0,\ I(0) = I_0,\ I_h(0) = I_{h0}$ and $R(0) = R_0$.

4. Construction of Algorithm

For the general solution of the consider Ebola model (3.1), we will perform some steps:

Step 1:
First of all the first derivative of $S(t),\ E(t),\ I(t),\ I_h(t)$ and $R(t)$ is given as

$$\begin{cases} S'(t_0) = \mu N - \frac{\beta(t)SI}{N} - \mu S, \\ E'(t_0) = \frac{\beta(t)SI}{N} - (\mu+\delta+\lambda)E, \\ I'(t_0) = \delta E - (\gamma+\alpha+\mu)I, \\ I_h'(t_0) = \lambda E + \alpha I - (\omega+\mu)I_h, \\ R' = \gamma I + \omega I_h - \mu R. \end{cases} \tag{4.1}$$

Step 2:
We compute the 2nd derivative of $S(t), E(t), I(t), I_h(t)$ and $R(t)$ as:

$$\begin{cases} S''(t_0) = -\frac{\beta(t)}{N}(SI' + S'I) - \mu S', \\ E''(t_0) = \frac{\beta(t)}{N}(SI' + IS') - (\mu+\delta+\lambda)E', \\ I''(t_0) = \delta E' - (\gamma+\alpha+\mu)I', \\ I_h''(t_0) = \lambda E' + \alpha I' - (\omega+\mu)I_h', \\ R''(t_0) = \gamma I' + \omega I_H' - \mu R'. \end{cases} \tag{4.2}$$

Step 3:
We compute the 3rd derivative of $S(t), E(t), I(t)\ I_h(t)$ and $R(t)$ as:

$$\begin{cases} S'''(t_0) = -\frac{\beta(t)}{N}(SI'' + 2S'I' + S''I) - \mu S'', \\ E'''(t_0) = \frac{\beta(t)}{N}(SI'' + 2S'I' + S''I) - (\mu+\delta+\lambda)E'', \\ I'''(t_0) = \delta E'' - (\gamma+\alpha+\mu)I'', \\ I_h'''(t_0) = \lambda E'' + \alpha I'' - (\omega+\mu)I_h'', \\ R'''(t_0) = \gamma I'' + \omega I_h'' - \mu R''. \end{cases} \tag{4.3}$$

Step 4:

We compute the 4th derivative of $S(t), E(t), I(t)\ I_h(t)$ and $R(t)$ as:

$$\begin{cases} S^{iv}(t_0) = -\frac{\beta(t)}{N}(SI''' + 3S'I'' + 3S''I' + S'''I) - \mu S''', \\ E^{iv}(t_0) = \frac{\beta(t)}{N}(SI''' + 3S''I' + 3S'I'' + S'''I) - (\mu+\delta+\lambda)E''', \\ I^{iv}(t_0) = \delta E''' - (\gamma+\alpha+\mu)I''', \\ I_h^{iv}(t_0) = \lambda E''' + \alpha I''' - (\omega+\mu)I_h''', \\ R^{iv}(t_0) = \gamma I''' + \omega I_h''' - \mu R'''. \end{cases} \tag{4.4}$$

Now the solution for the first few terms is given by

$$\begin{cases} S(t) = S(t_0) + tS'(t_0) + \frac{t^2}{2!}S''(t_0) + \frac{t^3}{3!}S'''(t_0) + \cdots, \\ E(t) = E(t_0) + tE'(t_0) + \frac{t^2}{2!}E''(t_0) + \frac{t^3}{3!}E'''(t_0) + \cdots, \\ I(t) = I(t_0) + tI'(t_0) + \frac{t^2}{2!}I''(t_0) + \frac{t^3}{3!}I'''(t_0) + \cdots, \\ I_h(t) = I_h(t_0) + tI_h'(t_0) + \frac{t^2}{2!}I_h''(t_0) + \frac{t^3}{3!}I_h'''(t_0) + \cdots, \\ R(t) = R(t_0) + tR'(t_0) + \frac{t^2}{2!}R''(t_0) + \frac{t^3}{3!}R'''(t_0) + \cdots. \end{cases} \tag{4.5}$$

Substituting the values of equations (4.1), (4.2), (4.3), (4.4) in (4.5).

$$\begin{cases} S(t) = S(t_0) + t\Big(\mu N - \frac{\beta(t)SI}{N} - \mu S\Big) + \frac{t^2}{2!}\Big(-\frac{\beta(t)}{N}(SI + S'I) - \mu S'\Big) \\ +\frac{t^3}{3!}\Big(-\frac{\beta(t)}{N}(SI'' + 2S'I' + S''I) - \mu S''\Big) \\ +\frac{t^4}{4!}\Big(-\frac{\beta(t)}{N}(SI''' + 3S'I'' + 3S''I' + S'''I) - \mu S'''\Big) + \cdots. \end{cases} \tag{4.6}$$

$$\begin{cases} E(t) = E(t_0) + t\Big(\frac{\beta(t)SI}{N} - (\mu+\delta+\lambda)E\Big) \\ +\frac{t^2}{2!}\Big(\frac{\beta(t)}{N}(SI' + IS')\Big) - (\mu+\delta+\lambda)E'\Big) \\ +\frac{t^3}{3!}\Big(\frac{\beta(t)}{N}(SI'' + 2S'I' + S''I) - (\mu+\delta+\lambda)E''\Big) \\ +\frac{t^4}{4!}\Big(\frac{\beta(t)}{N}(SI''' + 3S''I' + 3S'I'' + S'''I) - (\mu+\delta+\lambda)E'''\Big) + \cdots, \end{cases} \tag{4.7}$$

$$\begin{cases} I(t) = I(t_0) + t\Big(\delta E - (\gamma+\alpha+\mu)I\Big) + \frac{t^2}{2!}\Big(\delta E' - (\gamma+\alpha+\mu)I'\Big) \\ +\frac{t^3}{3!}\Big(\delta E'' - (\gamma+\alpha+\mu)I''\Big) \\ +\frac{t^4}{4!}\Big(\delta E''' - (\gamma+\alpha+\mu)I'''\Big) + \cdots, \end{cases} \tag{4.8}$$

$$\begin{cases} I_h(t) = I_h(t_0) + t\Big(\lambda E + \alpha I - (\omega+\mu)I_h\Big) + \frac{t^2}{2!}\Big(\delta E' - (\gamma+\alpha+\mu)I'\Big) \\ +\frac{t^3}{3!}\Big(\lambda E'' + \alpha I'' - (\omega+\mu)I_h''\Big) \\ +\frac{t^4}{4!}\Big(\lambda E''' + \alpha I''' - (\omega+\mu)I_h'''\Big) + \cdots, \end{cases} \tag{4.9}$$

$$\begin{cases} R(t) = R(t_0) + t\Big(\gamma I + \omega I_h - \mu R\Big) + \frac{t^2}{2!}\Big(\gamma I' + \omega I_h' - \mu R'\Big) \\ +\frac{t^3}{3!}\Big(\gamma I'' + \omega I_h{}'' - \mu R''\Big) \\ +\frac{t^4}{4!}\Big(\gamma I''' + \omega I_h''' - \mu R'''\Big) \cdots. \end{cases} \tag{4.10}$$

5. Numerical Discussion

To present the concerned approximate solutions computed above of the model under consideration, we recall some numerical values for the parameters in the given table 1. Based on reported data the initial condition is set as

$$\Big(S(t_0),\ E(t_0),\ I_h(t_0),\ I(t_0),\ R(t_0)\Big) = (460, 10, 12, 5, 0).$$

After putting the numerical values, we obtained the following results.

$$\begin{cases} S'(t_0) = 0.436456, \\ E'(t_0) = -12.0713955, \\ I'(t_0) = 2.13, \\ I_h'(t_0) = 9.2875, \\ R'(t_0) = 0.185, \end{cases} \tag{5.1}$$

$$\begin{cases} S''(t_0) = -0.0064321, \\ E''(t_0) = 0.0006438, \\ I''(t_0) = -7.989567, \\ I_h''(t_0) = 9.104803125, \\ R''(t_0) = 0.0606625, \end{cases} \tag{5.2}$$

$$\begin{cases} S'''(t_0) = 0.002409895, \\ E'''(t_0) = -0.002398614, \\ I'''(t_0) = 2.1775046, \\ I_h'''(t_0) = -2.38392741, \\ R'''(t_0) = -0.240846385, \end{cases} \tag{5.3}$$

$$\begin{cases} S^{iv}(t_0) = -0.000689414616, \\ E^{iv}(t_0) = -0.000132733556, \\ I^{iv}(t_0) = 3.43421855844, \\ I_h^{iv}(t_0) = 0.644133951, \\ R^{iv}(t_0) = 0.04495850512, \end{cases} \tag{5.4}$$

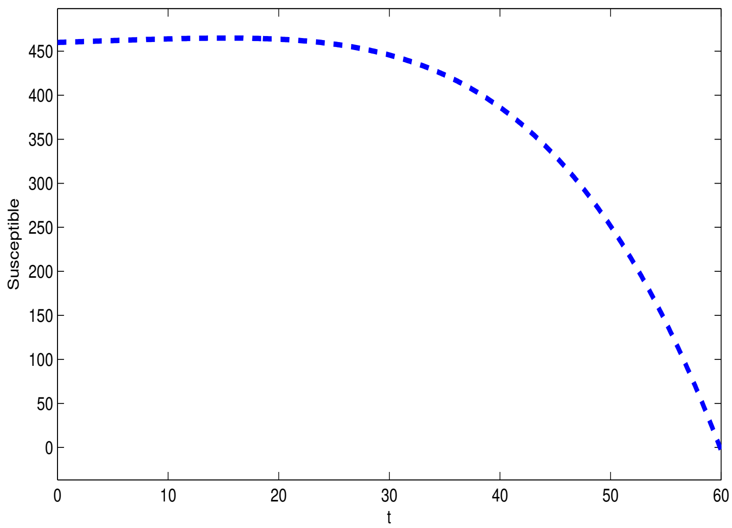

Figure 1: The Dynamical behavior of Susceptible Class.

$$\begin{cases} S^{\nu}(t_0) = -0.00104611315, \\ E^{\nu}(t_0) = 0.01052144, \\ I^{\nu}(t_0) = 2.43421855844, \\ I_h^{\nu}(t_0) = 0.133951, \\ R^{\nu}(t_0) = 0.3745925, \end{cases} \tag{5.5}$$

and so on. In this way the other terms may be computed. The series solution can write as:

$$\begin{cases} S(t) = \sum_{k=0}^{\infty} \frac{S^k(t_0)}{k!} t^k, \\ E(t) = \sum_{k=0}^{\infty} \frac{E^k(t_0)}{k!} t^k, \\ I(t) = \sum_{k=0}^{\infty} \frac{I^k(t_0)}{k!} t^k, \\ I_h(t) = \sum_{k=0}^{\infty} \frac{I_h^k(t_0)}{k!} t^k, \\ R(t) = \sum_{k=0}^{\infty} \frac{R^k(t_0)}{k!} t^k. \end{cases} \tag{5.6}$$

5.0.1. Figures and Tables

By using MATLAB we plot the solution as shown in Figures 1-5.

In Figures 1-5, we have provided graphical representation of different classes for the proposed model. We see that the Taylors series is a powerful technique for finding the numerical solution of the non linear problem. In Figure 3, we see that increase in infected class occurred but due to prevention and treatment procedure there is also increase in recovered class shown in Figure 5.

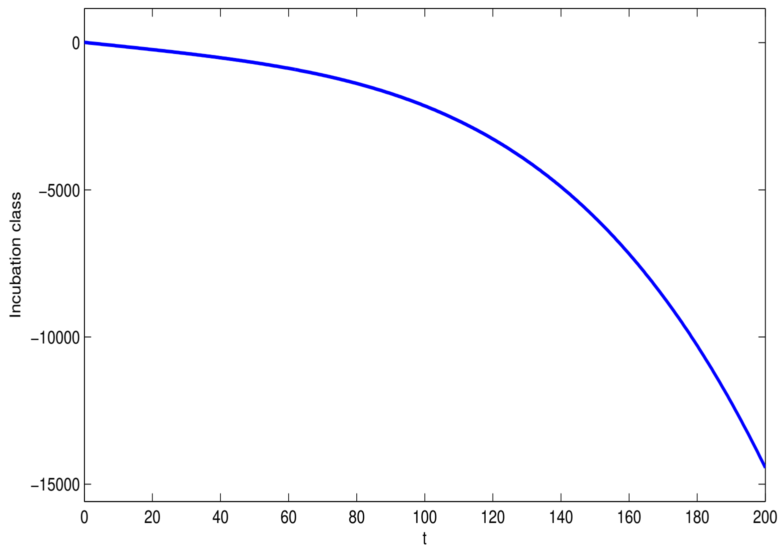

Figure 2: The Dynamical behavior of Incubation Class.

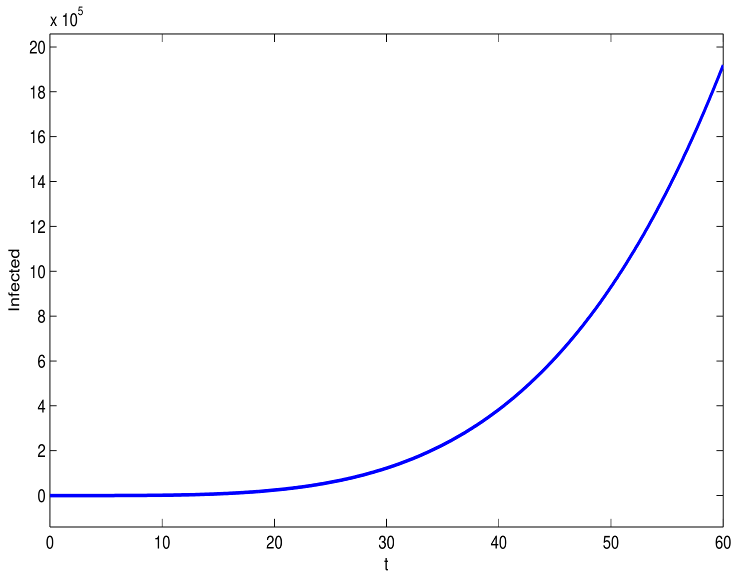

Figure 3: The Dynamical behavior of Infected Class.

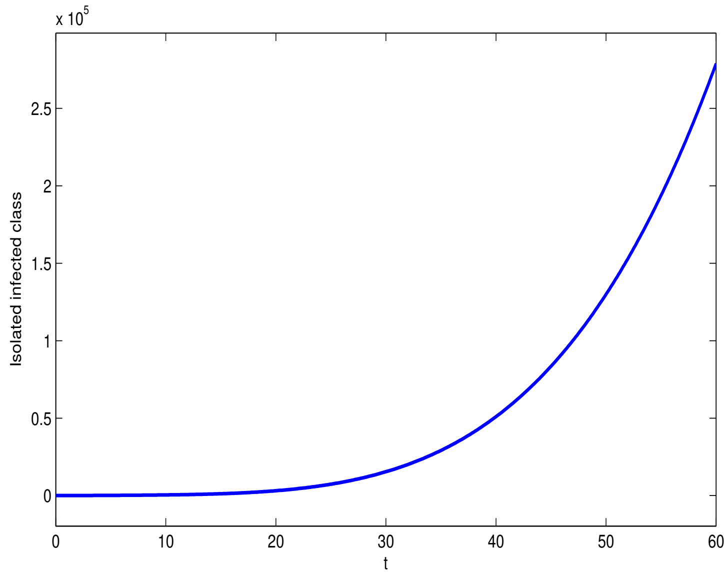

Figure 4: The Dynamical behavior of Isolated infected Class.

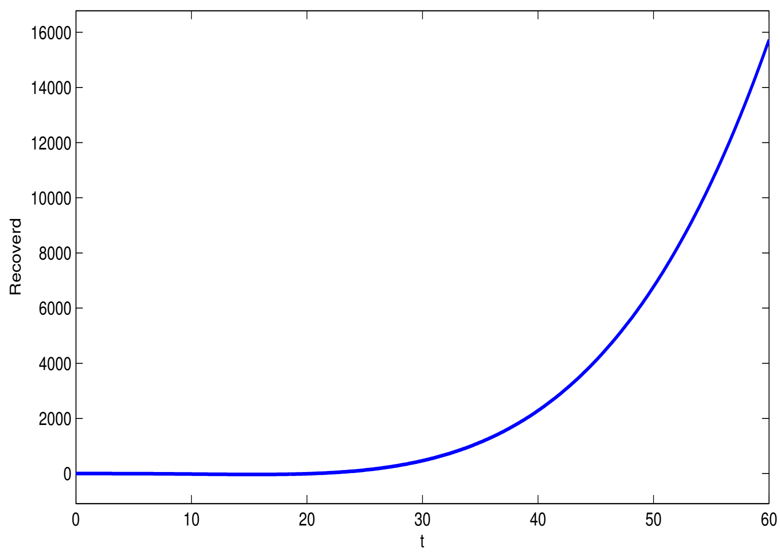

Figure 5: The Dynamical behavior of Recovered Class.

Parameters	Definition	Est: mean value	Source
μ	death rate	0.00318 per day	[22]
β	transmission rate from I and $\mathcal{S}$	0.0175	[22]
δ^{-1}	Duration of stay in the compartment E	0.540	[22]
γ^{-1}	Duration of stay in the compartment I	0.025	[22]
ω^{-1}	Duration an I transfer from I_h to death	0.250	[22]
λ	Proportion rate of E into isolated compartment	1.75	[22]
α	Proportion rate of I into isolated compartment	1/15 per day	[22]

Table 1: Description of the parameters of the Model (3.1).

6. Some Explanation and Concluding Remarks

In this manuscript, we have studied a four compartmental mathematical model of Ebola virus. The five compartments are of susceptible humans, the incubation period with which the disease, the number of infected humans, the isolation infectious human and infected humans who recovered at time t. By TSM some approximate analytical results are determined. Then using some real values for the parameters and initial data, we compute few terms approximate solutions corresponding to different compartment. With the help of MATLAB, we also plot our approximate solutions for different compartment graphically. The derived results indicate that the approximate solution by using Taylor's series method are more reliable and accurate. And does not required any small or large parameter assumption and accuracy of technique increases with increasing the order of approximation. Also in future this work will be interesting in the fractional case.

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