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## Dynamics behavior of solitons solutions of Chen-Lee-Liu equation using analytical techniques

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• Received: 09 May 2022

• Accepted: 15 June 2022

• Published Online: 28 June 2022

### Abstract

The main concern of the this article is to study the non-linear Chen-Lee-Liu equation which describes the motion of waves in shallow water. For this purpose two analytical techniques namely, the Sardar-Subequation method and the new extended hyperbolic function method are utilized. Also, we established the idea of the construction of solitons solutions of non-linear evolution equations which are rising in fluid dynamics, nonlinear optics, mathematical biological models, mechanics, waves theory, quantum mechanics. Acquired solutions are demonstrated graphically to reveal the dynamics behavior of solitons solutions. It hoped that the established solutions can be used to enrich the dynamic behaviors of Chen-Lee-Liu equation. Further, these solutions disclose that our techniques are up-to-date, suitable and straightforward.

Keywords: Chen-Lee-Liu equation, Sardar-subequation method, Solitons solutions.

### 1. Introduction

Searching for the analytical solutions of nonlinear partial differential equations play a vital role in physics and engineering. Novel ideas in physical sciences such as, fluid dynamic, computational physics, astrophysics, mathematical physics, medical physics and biological physics are explained by mathematical modeling [1]. Most of the real life phenomena are explained through waves and solitons. Various meaningful techniques have been established for solving wave structures of nonlinear partial differential equations, including the expansion approach [2], improved sub-equation scheme [3], extended Jacobi elliptic function expansion method [4], modified fractional reduced differential transform method [5], singular manifold method [6], modified  $(G'/G)$ -expansion approach [7], fractional reduced differential transform method [8], Sine-Gordon expansion method [9], extended modified mapping method [10], iterative method [11], extended trial equation method [12], simplest equation method [13], F-expansion method homo separation analysis method [14], extended mapping method [15], modified simple equation method [16], modified extended mapping method [17], reduced differential transform scheme

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[18], extended direct algebraic method [19], functional variable method [20], Darcy's law rule [21], Kudryashov method [22], auxiliary equation scheme [23], Sine-cosine approach [24]. In future, we will use the fractional calculus to construct analytical solutions of non-linear models under the Caputo-Fabrizio fractional derivative [25], fractional Atangana-Baleanu operator [26], Atangana, Baleanu and Caputo fractional order derivative [27]. In this work, our aim is to construct and analyze the novel solitons solutions using the Sardar-subequation method [28], and the new extended hyperbolic function method [29]. These solutions have many applications and are useful in different areas of physics, engineering and other fields of applied sciences. These general solutions can also provide a useful help for researchers to study and understand the physical interpretation of systems.

This work is structured as, the governing equation is present in section 2, The description of the Sardar-subequation method and its application on the perturbed Chen-Lee-Liu equation is discussed in section 3. Section 4 deals description of the the new extended hyperbolic function method and its application on the perturbed Chen-Lee-Liu equation. Results and discussion is presented in section 5 while Conclusion is written in section 6.

## 2. The perturbed Chen-Lee-Liu equation

Let the perturbed Chen-Lee-Liu equation [30],

$$i\Omega_t + \alpha\Omega_{xx} + i\beta|\Omega|^2\Omega_x = i\left[\gamma\Omega_x + \mu(|\Omega|^{2n}\Omega)_x + \delta(|\Omega|^{2n})_x\Omega\right], \quad (2.1)$$

where  $\Omega$  is the propagating disturbance,  $\gamma$ ,  $\delta$  and  $\mu$  are coefficients of inter model dispersion, nonlinear dispersion and self-steeping, respectively.  $\alpha$  and  $\beta$  are the coefficients of group velocity dispersion and non-linearity. In Eq. (2.1)  $n$  is the density of the wave function. The perturbed Chen-Lee-Liu equation demonstrate physical behavior of solitons and also has applications in optoelectronics, solitons cooling, optics, fluid dynamics and metamaterial [30]. Then Eq. (2.1) becomes,

$$i\Omega_t + \alpha\Omega_{xx} + i\beta|\Omega|^2\Omega_x = i\left[\gamma\Omega_x + \mu(|\Omega|^2\Omega)_x + \delta(|\Omega|^2)_x\Omega\right], \quad (2.2)$$

where  $n = 1$ . We construct a diversity of solitons by using the the Sardar-Subequation method and the new EHFm.

$$\Omega(x, t) = P(\eta) \exp^{i\phi}, \quad \eta = x - \lambda t, \quad \phi = -kx + \omega t + \xi \quad (2.3)$$

Using (2.3) into (2.2), gets an ODE as follows

$$i\lambda P' - \omega P + \alpha P'' - 2ik\alpha P' - k^2\alpha P + i\beta P'P^2 + k\beta P^3 - i\gamma P' - k\gamma P - 3i\mu P'P^2 - k\mu P^3 - 2i\delta P'P^2 = 0, \quad (2.4)$$

Eq. (2.4) can be split into real and imaginary parts, respectively.

$$k(\beta - \mu)P^3 + \alpha P'' - (\omega + \alpha k^2 + \gamma k)P = 0, \quad (2.5)$$

$$(\beta - 3\mu - 2\delta)P'P^2 - (\lambda + 2\alpha k + \gamma)P' = 0. \quad (2.6)$$

We get  $\beta = 3\mu + 2\delta$  and  $\lambda = -(\gamma + 2\alpha k)$  by setting the components of imaginary part equal to zero. Under the above mentioned two constraints, the real part (2.5) becomes,

$$2k(\lambda + \mu)P^3 + \alpha P'' - (\omega + \alpha k^2 + \gamma k)P = 0. \quad (2.7)$$

### 3. The Sardar-subequation Method

Consider the PDE

$$W(\Omega, \Omega_t, \Omega_x, \Omega_{xx}, \Omega_{xt}, \Omega_{tt}, \Omega_{tx}, \dots) = 0, \tag{3.1}$$

where  $\Omega = \Omega(x, t)$  is a function.

$$\Omega(x, t) = P(\eta) \exp^{i\phi}, \quad \eta = x - \lambda t, \quad \phi = -kx + \omega t + \xi, \tag{3.2}$$

where  $v$  is non-zero constant. By applying the above transformation, given below ODE is obtained

$$V(P, P', P'', \dots) = 0, \tag{3.3}$$

in which  $P = P(\eta), P' = \frac{dP}{d\eta}, P'' = \frac{d^2P}{d\eta^2}, \dots$

Assume the solution of (3.3) is in the form

$$P(\eta) = \sum_{k=0}^M \Lambda_k Q^k(\eta), \tag{3.4}$$

where  $\Lambda_k$  ( $0 \leq k \leq M$ ) are constants.

$$(Q'(\eta))^2 = C + A Q^2(\eta) + Q^4(\eta), \tag{3.5}$$

where  $C$  and  $A$  are real constants and (3.5) yields the sets of solutions as

**Case 1:** When  $A > 0$  and  $C = 0$ , then

$$Q_1^\pm(\eta) = \pm \sqrt{-Apq} \operatorname{sech}_{pq}(\sqrt{A}\eta),$$

$$Q_2^\pm(\eta) = \pm \sqrt{Apq} \operatorname{csch}_{pq}(\sqrt{A}\eta),$$

where

$$\operatorname{sech}_{pq}(\eta) = \frac{2}{pe^\eta + qe^{-\eta}}, \quad \operatorname{csch}_{pq}(\eta) = \frac{2}{pe^\eta - qe^{-\eta}}.$$

**Case 2:** When  $A < 0$  and  $C = 0$ , then

$$Q_3^\pm(\eta) = \pm \sqrt{-Apq} \operatorname{sec}_{pq}(\sqrt{-A}\eta),$$

$$Q_4^\pm(\eta) = \pm \sqrt{-Apq} \operatorname{csc}_{pq}(\sqrt{-A}\eta),$$

where

$$\sec_{pq}(\eta) = \frac{2}{pe^{\eta}+qe^{-\eta}}, \quad \csc_{pq}(\eta) = \frac{2i}{pe^{\eta}-qe^{-\eta}}.$$

**Case 3:** When  $A < 0$  and  $C = \frac{a^2}{4b}$ , then

$$Q_5^{\pm}(\eta) = \pm \sqrt{-\frac{A}{2}} \tanh_{pq}\left(\sqrt{-\frac{A}{2}} \eta\right),$$

$$Q_6^{\pm}(\eta) = \pm \sqrt{-\frac{A}{2}} \coth_{pq}\left(\sqrt{-\frac{A}{2}} \eta\right),$$

$$Q_7^{\pm}(\eta) = \pm \sqrt{-\frac{A}{2}} \left( \tanh_{pq}(\sqrt{-2A} \eta) \pm i\sqrt{pq} \operatorname{sech}_{pq}(\sqrt{-2A} \eta) \right),$$

$$Q_8^{\pm}(\eta) = \pm \sqrt{-\frac{A}{2}} \left( \coth_{pq}(\sqrt{-2A} \eta) \pm \sqrt{pq} \operatorname{csch}_{pq}(\sqrt{-2A} \eta) \right),$$

$$Q_9^{\pm}(\eta) = \pm \sqrt{-\frac{A}{8}} \left( \tanh_{pq}\left(\sqrt{-\frac{A}{8}} \eta\right) + \coth_{pq}\left(\sqrt{-\frac{A}{8}} \eta\right) \right),$$

where

$$\tanh_{pq}(\eta) = \frac{pe^{\eta}-qe^{-\eta}}{pe^{\eta}+qe^{-\eta}}, \quad \coth_{pq}(\eta) = \frac{pe^{\eta}+qe^{-\eta}}{pe^{\eta}-qe^{-\eta}}.$$

**Case 4:** When  $A > 0$  and  $C = \frac{a^2}{4}$ , then

$$Q_{10}^{\pm}(\eta) = \pm \sqrt{\frac{A}{2}} \tan_{pq}\left(\sqrt{\frac{A}{2}} \eta\right),$$

$$Q_{11}^{\pm}(\eta) = \pm \sqrt{\frac{A}{2}} \cot_{pq}\left(\sqrt{\frac{A}{2}} \eta\right),$$

$$Q_{12}^{\pm}(\eta) = \pm \sqrt{\frac{A}{2}} \left( \tan_{pq}(\sqrt{2A} \eta) \pm \sqrt{pq} \operatorname{sec}_{pq}(\sqrt{2A} \eta) \right),$$

$$Q_{13}^{\pm}(\eta) = \pm \sqrt{\frac{A}{2}} \left( \cot_{pq}(\sqrt{2A} \eta) \pm \sqrt{pq} \operatorname{csc}_{pq}(\sqrt{2A} \eta) \right),$$

$$Q_{14}^{\pm}(\eta) = \pm \sqrt{\frac{A}{8}} \left( \tan_{pq}\left(\sqrt{\frac{A}{8}} \eta\right) + \cot_{pq}\left(\sqrt{\frac{A}{8}} \eta\right) \right),$$

where

$$\tan_{pq}(\eta) = -i \frac{pe^{\eta}-qe^{-\eta}}{pe^{\eta}+qe^{-\eta}}, \quad \cot_{pq}(\eta) = i \frac{pe^{\eta}+qe^{-\eta}}{pe^{\eta}-qe^{-\eta}}.$$

3.1. Application of the Sardar-subequation Method

In order to find the solitons solutions of the perturbed Chen-Lee-Liu equation the the Sardar-subequation method is applied on (2.7). By balance rule on terms of  $P''$  and  $P^3$  in (2.7), yields  $M = 1$ , so (3.4) reduces to

$$P(\eta) = \Lambda_0 + \Lambda_1 Q(\eta), \tag{3.6}$$

where  $\Lambda_0, \Lambda_1$  are constants. Replacing the (3.6) into (2.7) and gets the set of equations in  $\Lambda_0, \Lambda_1, \alpha$ , and  $k$ . On solving the set of equations, we yields

$$\Lambda_0 = 0, \quad \Lambda_1 = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}},$$

$$\alpha = \frac{w + k\gamma}{\Lambda - k^2}. \tag{3.7}$$

**Case 1:** When  $\Lambda > 0$  and  $C = 0$ , then

$$Z_{1,1}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{-pq\Lambda} \operatorname{sech}_{pq}(\sqrt{\Lambda}(x - \lambda t)) \right) \times e^{i\phi}, \tag{3.8}$$

$$Z_{1,2}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{pq\Lambda} \operatorname{csch}_{pq}(\sqrt{\Lambda}(x - \lambda t)) \right) \times e^{i\phi}. \tag{3.9}$$

**Case 2:** When  $\Lambda < 0$  and  $C = 0$ , then

$$Z_{1,3}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{-pq\Lambda} \operatorname{sec}_{pq}(\sqrt{-\Lambda}(x - \lambda t)) \right) \times e^{i\phi}, \tag{3.10}$$

$$Z_{1,4}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{-pq\Lambda} \operatorname{csc}_{pq}(\sqrt{-\Lambda}(x - \lambda t)) \right) \times e^{i\phi}. \tag{3.11}$$

**Case 3:** When  $\Lambda < 0$  and  $C = \frac{a^2}{4b}$ , then

$$Z_{1,5}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{-\frac{\Lambda}{2}} \operatorname{tanh}_{pq}\left(\sqrt{-\frac{\Lambda}{2}}(x - \lambda t)\right) \right) \times e^{i\phi}, \tag{3.12}$$

$$Z_{1,6}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{-\frac{\Lambda}{2}} \operatorname{coth}_{pq}\left(\sqrt{-\frac{\Lambda}{2}}(x - \lambda t)\right) \right) \times e^{i\phi}, \tag{3.13}$$

$$Z_{1,7}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(\Lambda - k^2)(\delta + \mu)}} \left( \pm \sqrt{-\frac{\Lambda}{2}} \left( \operatorname{tanh}_{pq}(\sqrt{-2\Lambda}(x - \lambda t)) \right. \right. \\ \left. \left. \pm i\sqrt{pq} \operatorname{sech}_{pq}(\sqrt{-2\Lambda}(x - \lambda t)) \right) \right) \times e^{i\phi}, \tag{3.14}$$

$$Z_{1,8}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{-\frac{A}{2}} \left( \coth_{pq}(\sqrt{-2A}(x - \lambda t)) \pm \sqrt{pq} \operatorname{csch}_{pq}(\sqrt{-2A}(x - \lambda t)) \right) \right) \times e^{i\phi}, \quad (3.15)$$

$$Z_{1,9}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{-\frac{A}{8}} \left( \tanh_{pq}(\sqrt{-\frac{A}{8}}(x - \lambda t)) + \coth_{pq}(\sqrt{-\frac{A}{8}}(x - \lambda t)) \right) \right) \times e^{i\phi}. \quad (3.16)$$

**Case 4:** When  $A > 0$  and  $C = \frac{a^2}{4}$ , then

$$Z_{1,10}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{\frac{A}{2}} \tan_{pq}(\sqrt{\frac{A}{2}}(x - \lambda t)) \right) \times e^{i\phi}, \quad (3.17)$$

$$Z_{1,11}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{\frac{A}{2}} \cot_{pq}(\sqrt{\frac{A}{2}}(x - \lambda t)) \right) \times e^{i\phi}, \quad (3.18)$$

$$Z_{1,12}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{\frac{A}{2}} \left( \tan_{pq}(\sqrt{2A}(x - \lambda t)) \pm \sqrt{pq} \operatorname{sec}_{pq}(\sqrt{2A}(x - \lambda t)) \right) \right) \times e^{i\phi}, \quad (3.19)$$

$$Z_{1,13}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{\frac{A}{2}} \left( \cot_{pq}(\sqrt{2A}(x - \lambda t)) \pm \sqrt{pq} \operatorname{csc}_{pq}(\sqrt{2A}(x - \lambda t)) \right) \right) \times e^{i\phi}, \quad (3.20)$$

$$Z_{1,14}^{\pm}(x, t) = -\frac{\sqrt{-w - k\gamma}}{\sqrt{k(A - k^2)(\delta + \mu)}} \left( \pm \sqrt{\frac{A}{8}} \left( \tan_{pq}(\sqrt{\frac{A}{8}}(x - \lambda t)) + \cot_{pq}(\sqrt{\frac{A}{8}}(x - \lambda t)) \right) \right) \times e^{i\phi}. \quad (3.21)$$

#### 4. New Extended Hyperbolic Function Method

This scheme has two phases as follows

**Phase 1:** Consider the PDE (2.2), using wave transformation (2.3) into (2.2) then ODE (2.7) is obtained. Assume that given below is the solution of (2.7)

$$P(\eta) = \sum_{k=0}^M \Lambda_k Q^k(\eta), \quad (4.1)$$

where  $\Lambda_k (k = (1, 2, 3, \dots, M))$  are constants and given below solution admits the  $Q(\eta)$

$$\frac{dQ}{d\eta} = Q\sqrt{R + SQ^2}, \quad R, S \in \mathfrak{R}. \quad (4.2)$$

By balance principle on (2.7) the value of  $M$  is determined. Putting (4.1) into (2.7) with (4.2), obtained a set of equations. On solution the set of equations we obtained

**Set 1:** When  $R > 0$  and  $S > 0$ ,

$$Q(\eta) = -\sqrt{\frac{R}{S}} \operatorname{csch}(\sqrt{R}(\eta + \eta_0)). \quad (4.3)$$

**Set 2:** When  $R < 0$  and  $S > 0$ ,

$$Q(\eta) = \sqrt{\frac{-R}{S}} \operatorname{sec}(\sqrt{-R}(\eta + \eta_0)). \quad (4.4)$$

**Set 3:** When  $R > 0$  and  $S < 0$ ,

$$Q(\eta) = \sqrt{\frac{R}{-S}} \operatorname{sech}(\sqrt{-S}(\eta + \eta_0)). \quad (4.5)$$

**Set 4:** When  $R < 0$  and  $S > 0$ ,

$$Q(\eta) = \sqrt{\frac{-R}{S}} \operatorname{csc}(\sqrt{-S}(\eta + \eta_0)). \quad (4.6)$$

**Set 5:** When  $R > 0$  and  $S = 0$ ,

$$Q(\eta) = \exp(\sqrt{R}(\eta + \eta_0)). \quad (4.7)$$

**Set 6:** When  $R < 0$  and  $S = 0$ ,

$$Q(\eta) = \cos(\sqrt{-R}(\eta + \eta_0)) + i \sin(\sqrt{-R}(\eta + \eta_0)). \quad (4.8)$$

**Set 7:** When  $R = 0$  and  $S > 0$ ,

$$Q(\eta) = \pm \frac{1}{(\sqrt{S}(\eta + \eta_0))}. \quad (4.9)$$

**Set 8:** When  $R = 0$  and  $S < 0$ ,

$$Q(\eta) = \pm \frac{i}{(\sqrt{-S}(\eta + \eta_0))}. \quad (4.10)$$

**Phase 2:** Adopting the same procedure as phase 1,  $Q(\eta)$  admits the ODE given below

$$\frac{dQ}{d\eta} = R + SQ^2, \quad R, S \in \mathfrak{R}. \quad (4.11)$$

Putting (4.1) into (2.7) along with (4.11), makes a system of equations with the values of  $\Lambda_k (k = 1, 2, 3, \dots, M)$ . Let the (4.11) accepts the solutions as follows

**Set 1:** When  $RS > 0$ ,

$$Q(\eta) = \text{sgn}(R) \sqrt{\frac{R}{S}} \tan(\sqrt{RS}(\eta + \eta_0)). \quad (4.12)$$

**Set 2:** When  $RS > 0$ ,

$$Q(\eta) = -\text{sgn}(R) \sqrt{\frac{R}{S}} \cot(\sqrt{RS}(\eta + \eta_0)). \quad (4.13)$$

**Set 3:** When  $RS < 0$ ,

$$Q(\eta) = \text{sgn}(R) \sqrt{\frac{R}{-S}} \tanh(\sqrt{-RS}(\eta + \eta_0)). \quad (4.14)$$

**Set 4:** When  $RS < 0$ ,

$$Q(\eta) = \text{sgn}(R) \sqrt{\frac{R}{-S}} \coth(\sqrt{-RS}(\eta + \eta_0)). \quad (4.15)$$

**Set 5:** When  $R = 0$  and  $S > 0$ ,

$$Q(\eta) = -\frac{1}{S(\eta + \eta_0)}. \quad (4.16)$$

**Set 6:** When  $R \in \mathfrak{R}$  and  $S = 0$ ,

$$Q(\eta) = R(\eta + \eta_0). \quad (4.17)$$

**Note:**  $\text{sgn}$  is the famous sign functions.

## 5. Application of the New Extended Hyperbolic Function Method

**Phase 1:** In order to find the solution of the perturbed Chen-Lee-Liu equation (2.2) the new extended hyperbolic function method is applied. By balance principle on (2.7), yields  $M = 2$ , so (4.1) can be written as

$$P(\eta) = \Lambda_0 + \Lambda_1 Q(\eta), \quad (5.1)$$

where  $\Lambda_0$ , and  $\Lambda_1$  are constants. Putting (5.1) into (2.7) gets the set of equations in  $\Lambda_0$ ,  $\Lambda_1$ ,  $R$ , and  $k$ . On solution the set of equations, we acquire

$$\Lambda_0 = 0, \quad \Lambda_1 = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}},$$

$$R = \frac{w + k^2\alpha + k\gamma}{\alpha}, \quad S = S. \tag{5.2}$$

**Set 1:** When  $R > 0$  and  $S > 0$ ,

$$Z_1(x, t) = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( -\sqrt{\frac{w + k^2\alpha + k\gamma}{S\alpha}} \operatorname{csch}\left(\sqrt{\frac{w + k^2\alpha + k\gamma}{\alpha}}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \tag{5.3}$$

**Set 2:** When  $R < 0$  and  $S > 0$ ,

$$Z_2(x, t) = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \sqrt{-\frac{w + k^2\alpha + k\gamma}{S\alpha}} \operatorname{sec}\left(\sqrt{-\frac{w + k^2\alpha + k\gamma}{\alpha}}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \tag{5.4}$$

**Set 3:** When  $R > 0$  and  $S < 0$ ,

$$Z_3(x, t) = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \sqrt{-\frac{w + k^2\alpha + k\gamma}{S\alpha}} \operatorname{sech}\left(\sqrt{-S}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \tag{5.5}$$

**Set 4:** When  $R < 0$  and  $R < 0$ ,

$$Z_4(x, t) = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \sqrt{-\frac{w + k^2\alpha + k\gamma}{S\alpha}} \operatorname{csc}\left(\sqrt{-S}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \tag{5.6}$$

**Set 5:** When  $R = 0$  and  $S > 0$ ,

$$Z_5(x, t) = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \pm \frac{1}{(\sqrt{S}(\eta + \eta_0))} \right) \times e^{i\phi}. \tag{5.7}$$

**Set 6:** When  $R = 0$  and  $S < 0$ ,

$$Z_6(x, t) = \frac{\sqrt{S}\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \pm \frac{i}{(\sqrt{-S}(\eta + \eta_0))} \right) \times e^{i\phi}, \tag{5.8}$$

where  $\eta = x - \lambda t$  and  $\phi = -kx + \omega t + \xi$ ,

**Phase 2:**

Applying the balance principle on (2.7), gives  $M = 2$ , so (4.1) can be written as

$$P(\eta) = \Lambda_0 + \Lambda_1 Q(\eta), \quad (5.9)$$

where  $\Lambda_0$  and  $\Lambda_1$  are constants. Putting (5.9) into (2.7), we get a set of equations in  $\Lambda_0$ ,  $\Lambda_1$ , and  $R$ . On solution the set of equations, we acquire

$$\begin{aligned} \Lambda_0 = 0, \quad \Lambda_1 &= \frac{S\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}}, \\ R &= \frac{\omega + k(k\alpha + \gamma)}{2S\alpha}, \quad S = S. \end{aligned} \quad (5.10)$$

**Set 1:** When  $RS > 0$ ,

$$Z_7(x, t) = \frac{S\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \chi \sqrt{\frac{\omega + k(k\alpha + \gamma)}{2S^2\alpha}} \tan\left(\sqrt{\frac{\omega + k(k\alpha + \gamma)}{2\alpha}}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \quad (5.11)$$

**Set 2:** When  $RS > 0$ ,

$$Z_8(x, t) = \frac{S\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( -\chi \sqrt{\frac{\omega + k(k\alpha + \gamma)}{2S^2\alpha}} \cot\left(\sqrt{\frac{\omega + k(k\alpha + \gamma)}{2\alpha}}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \quad (5.12)$$

**Set 3:** When  $RS < 0$ ,

$$Z_9(x, t) = \frac{S\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \chi \sqrt{-\frac{\omega + k(k\alpha + \gamma)}{2S^2\alpha}} \tanh\left(\sqrt{-\frac{\omega + k(k\alpha + \gamma)}{2\alpha}}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \quad (5.13)$$

**Set 4:** When  $RS < 0$ ,

$$Z_{10}(x, t) = \frac{S\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( \chi \sqrt{-\frac{\omega + k(k\alpha + \gamma)}{2S^2\alpha}} \coth\left(\sqrt{-\frac{\omega + k(k\alpha + \gamma)}{2\alpha}}(\eta + \eta_0)\right) \right) \times e^{i\phi}. \quad (5.14)$$

**Set 5:** When  $R = 0$  and  $S > 0$ ,

$$Z_{11}(x, t) = \frac{S\sqrt{\alpha}}{\sqrt{-k(\lambda + \mu)}} \left( -\frac{1}{S(\eta + \eta_0)} \right) \times e^{i\phi}. \quad (5.15)$$

where  $\chi = \operatorname{sgn}\left(\frac{\omega + k(k\alpha + \gamma)}{2S\alpha}\right)$ ,  $\eta = x - \lambda t$  and  $\phi = -kx + \omega t + \xi$ .

### 6. Results and Discussions

We have successfully achieved various solitons solutions as well as trigonometric and hyperbolic function solutions for the perturbed the perturbed Chen-Lee-Liu equation equation using the the Sardar-subequation method and new extended hyperbolic function method. These analytic approaches are considered as latest techniques in this area and have not applied to this equation earlier. These acquired results are widely applicable in communication to carry energy because solitons have the capability to cover long spaces without reduction and without altering their shapes. In this paper, we only plotted specific solutions in the form of 2-D and 3-D graphs to observe the physical behavior of this model. Figures 1-7 demonstrate the solutions of equation (1) with the aid of free parameters. Figures 1-2 express the solutions of (3.8) and (3.12) which are bright and dark solitons, respectively. Figures 3-4 express the solutions of (3.14) and (3.21) which are combined dark-bright and mixed periodic-singular solitons, respectively. While Figures 5-6 demonstrate the solutions of (5.3) and (5.5) which are singular and bright solitons, respectively. Also, figure 7 denote the solutions of (5.13) that is dark soliton.

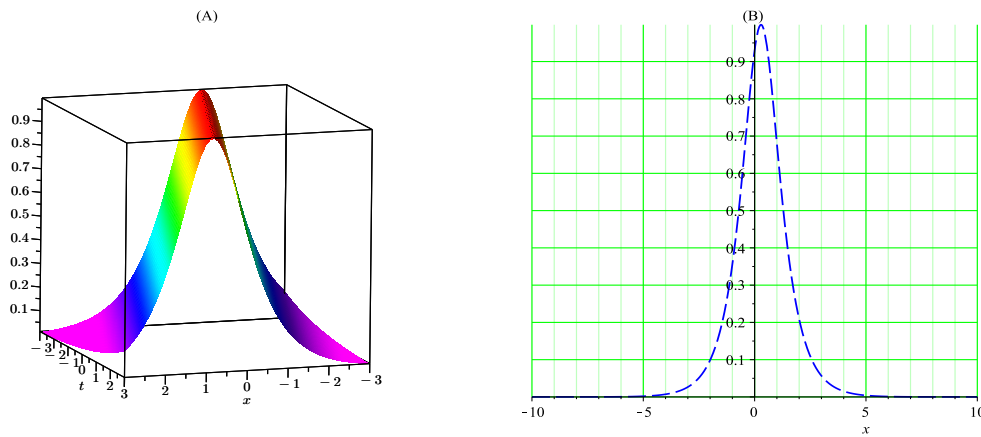


Figure 1: (A) 3D graph of (3.8) with  $w = 0.5$ ,  $k = 0.78$ ,  $\gamma = 0.3$ ,  $A = 2.6$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.75$ ,  $\delta = 0.7$ ,  $\mu = 1.2$ ,  $\lambda = 0.65$ ,  $\xi = 0.65$ . (B) 2D representation of (3.8) with  $t = 1$ .

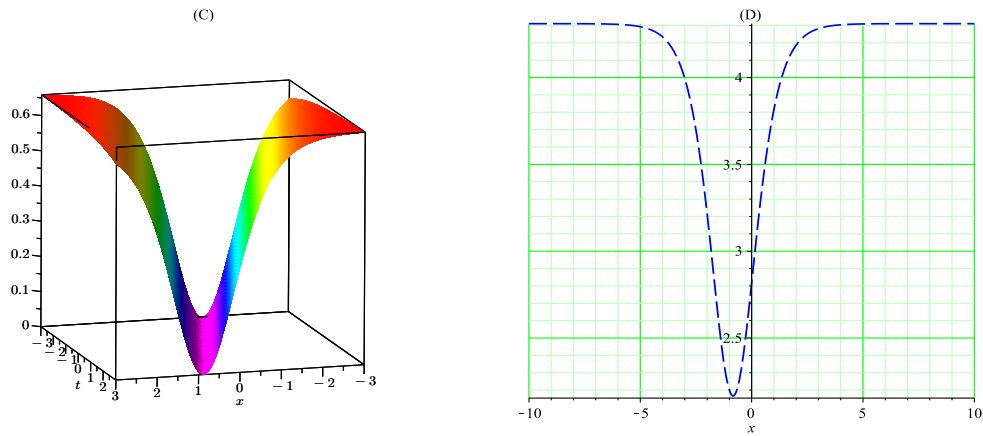


Figure 2: (C) 3D graph of (3.12) with  $w = 0.65$ ,  $k = 0.98$ ,  $\gamma = 0.32$ ,  $\Lambda = 2.5$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.55$ ,  $\delta = 0.71$ ,  $\mu = 1.23$ ,  $\lambda = 0.75$ ,  $\xi = 0.67$ . (D) 2D representation of (3.12) with  $t = 1$ .

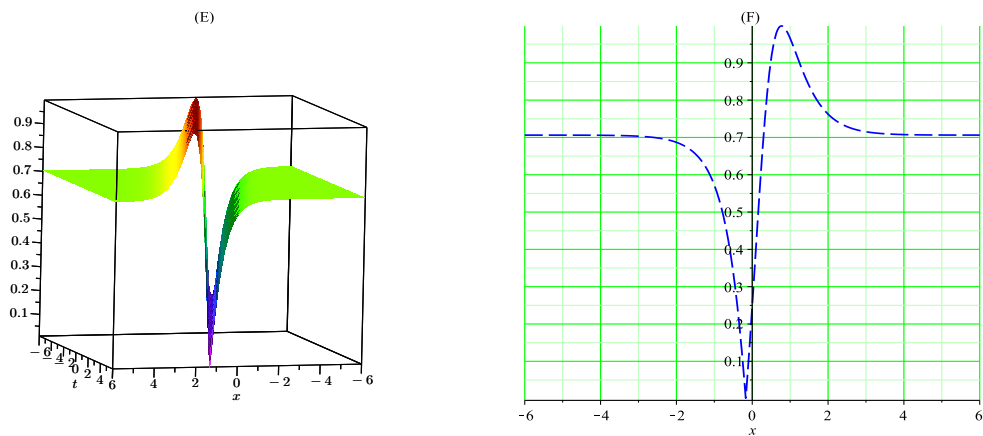


Figure 3: (E) 3D graph of (3.14) with  $w = 0.3$ ,  $k = 0.53$ ,  $\gamma = 0.34$ ,  $\Lambda = 2.4$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.45$ ,  $\delta = 0.65$ ,  $\mu = 1.3$ ,  $\lambda = 0.65$ ,  $\xi = 0.60$ . (F) 2D representation of (3.14) with  $t = 1$ .

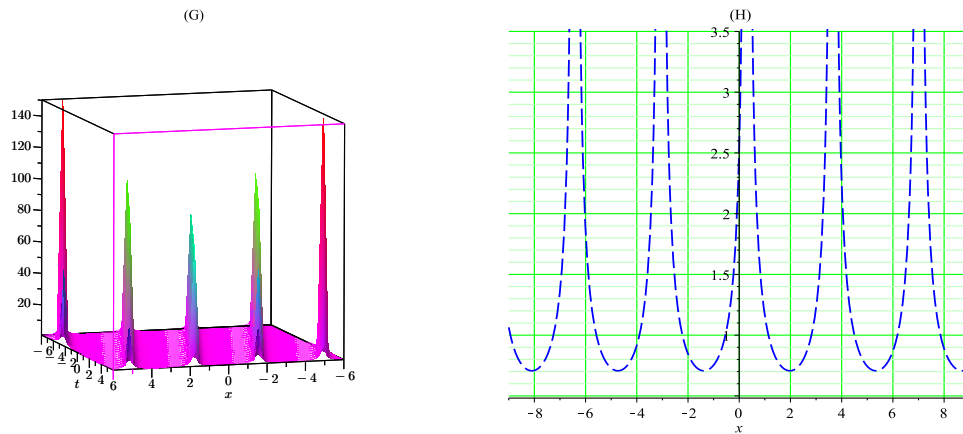


Figure 4: (G) 3D graph of (3.21) with  $w = 0.49$ ,  $k = 0.68$ ,  $\gamma = 0.43$ ,  $\Lambda = 2.5$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.65$ ,  $\delta = 0.8$ ,  $\mu = 1.3$ ,  $\lambda = 1.1$ ,  $\xi = 0.74$ . (H) 2D representation of (3.21) with  $t = 1$ .

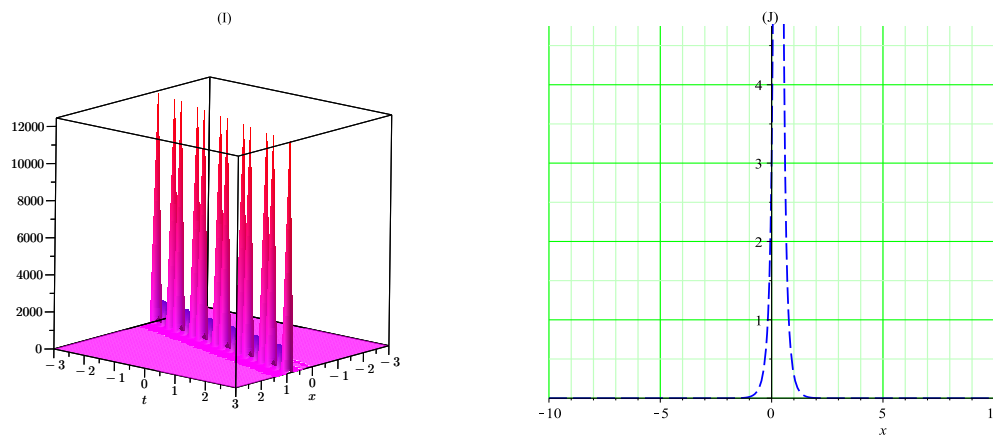


Figure 5: (I) 3D graph of (5.3) with  $S = 0.52$ ,  $w = 0.75$ ,  $k = 0.78$ ,  $\gamma = 0.4$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.85$ ,  $\delta = 0.67$ ,  $\mu = 1.4$ ,  $\lambda = 0.55$ ,  $\xi = 0.63$ . (J) 2D representation of (5.3) with  $t = 1$ .

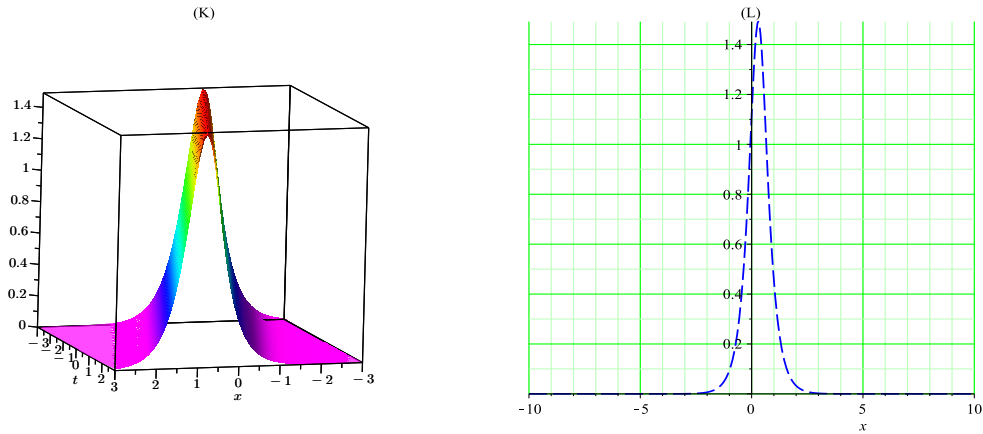


Figure 6: (K) 3D graph of (5.5) with  $S = 0.6$ ,  $w = 0.75$ ,  $k = 0.88$ ,  $\gamma = 0.43$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.85$ ,  $\delta = 0.71$ ,  $\mu = 1.3$ ,  $\lambda = 1.85$ ,  $\xi = 0.69$ . (L) 2D representation of (5.5) with  $t = 1$ .

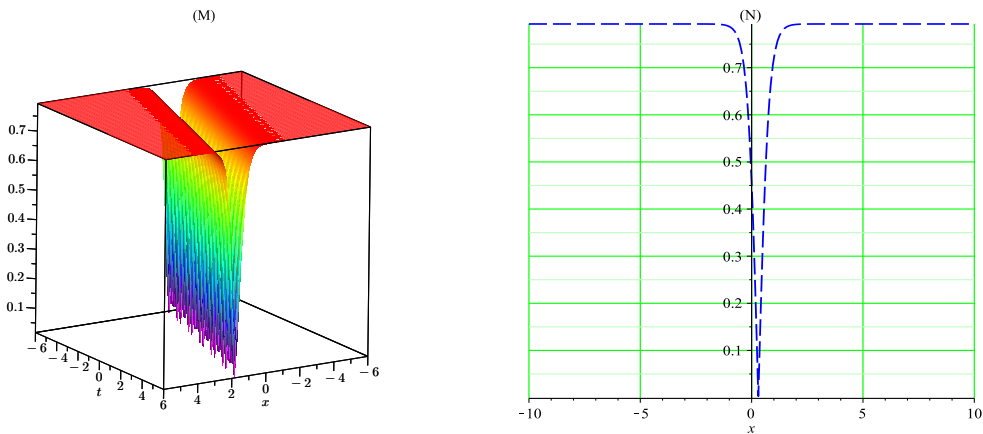


Figure 7: (M) 3D graph of (5.13) with  $S = 0.7$ ,  $w = 0.75$ ,  $k = 0.8$ ,  $\gamma = 0.4$ ,  $p = 1$ ,  $q = 1$ ,  $\alpha = 0.65$ ,  $\delta = 0.65$ ,  $\mu = 1.2$ ,  $\lambda = 0.55$ ,  $\xi = 0.67$ . (N) 2D representation of (5.13) with  $t = 1$ .

## 7. Conclusion

In this paper, we have successfully constructed the solitons solutions of a well-known non-linear perturbed Chen-Lee-Liu equation by using Sardar-subequation method and the new extended hyperbolic function method. The obtained solutions comprise rational, exponential, trigonometric, hyperbolic, periodic, dark, singular, bright and dark-bright combo solitons. Some appropriate values are taken for involved free parameters to demonstrate the graphical behavior of acquired solutions. Dynamics behavior of these solutions is discussed in detail, and physical properties of the current model. These achieved solutions have significance in various fields of non-linear sciences such as mathematical physics, optical fibers, fluid dynamics, pulse propagation, engineering, and many more. From results, we can conclude that the current methods are suitable, beneficial and skillful for getting

the solitons solutions of such kind of models.

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